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RESEARCH ARTICLE

Applications of the single-port linear Thevenin theorem for focused and efficient analysis of a sub-network connected with a large existing pipe network

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ABSTRACT
An existing water distribution network (WDN) may need to be expanded by adding a sub-network for the newly developed areas. The size of the problem becomes larger when the stochastic nature of domestic demands, optimal design and layouts, control, and operation of various hydraulic components are considered. In this study, the single-port Thevenin theorem used in electrical circuits is applied to reduce a large WDN with its equivalent network consisting of a single source and a single pipe. The equivalent network is then attached to a sub-network for focused analysis. The accuracy and robustness of the proposed network reduction procedure are investigated on realistic WDNs for various sub-network demands using steady and extended period simulations. A simplified approach is also presented to achieve the same objective but constrained by the level of accuracy. Hydraulic engineers can use the proposed methodology as an efficient network reduction tool.

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Water distribution network analysis; stochastic analysis; extended period simulation; Thevenin theorem

Introduction

The urban population is reported to increase by 185% by 2030 from circa 2000 (Seto, Güneralp, and Hutyra 2012) mainly due to migration. This increasing trend in population migration leads to the rapid expansion of cities all over the world. Providing water and maintaining water security in the expanding cities are quite challenging and expensive tasks (Allan, Kenway, and Head 2018). It may be required to expand the existing water distribution network (WDN) to the newly developed areas to meet extra water demand. However, it is costly to replace a large part of the existing WDN with a new system to cater to the additional demand of the expanded city limits. Swamee and Sharma (1990) reported that WDNs are, in general, designed for a pre-decided time span called the design period (that varies from 20 to 40 years), but the working life of pipelines can vary from 60 to 120 years. Swamee and Sharma (2008) suggested that reorganizing the existing WDN for the increased demand is an economical solution if the maximum size of the newly connected area is less than 20–25% of the existing WDN. Sometimes, depending upon the ground condition, reorganization might avoid modification of the existing WDN, but only involve the connection of a sub-network at a predetermined node of the existing pipe links. In this context, analysis of the sub-network for reliability and optimal design can become time-consuming, especially when the sub-network is connected to a large existing WDN. Under this circumstance, a pipe network analysis tool that focuses primarily on the sub-network can be handy for hydraulic engineers without investing much computation time in the already designed main network.

Agrawal, Gupta, and Bhave (2007) proposed a method for the reliability-based strengthening and expansion of WDNs based on the marginal capacity factor ratio to the marginal increase in cost. Todini (2000) developed multi-objective based approaches to design expansion and rehabilitation of an existing WDN. Neelakantan et al. (2014) proposed a simulation-optimization model for cost-effective upgrade and expansion of existing WDNs. It should be noted that in the analysis of all the above WDN expansion related studies, the large existing networks along with the sub-network for the expanded area were repeatedly simulated using hydraulic models, resulting in a high computational burden. Moreover, the computation time will further increase if the sub-network were to be designed for stochastic demands in the expanded area.

In network design approaches, the total demand in a WDN is assumed to be deterministic, while in reality, the total demand at a location is variable in terms of both daily and seasonal fluctuations. Gargano et al. (2016) proposed the Overall Pulse (OP) model, which allows the overall domestic demand generation as displayed at the house water meter. Brentan et al. (2018) proposed different methods to generate synthetic time series of water demand from observed data. Nodal demand and pressure can also change due to deterioration of flow capacity caused by the aging of pipes (Piller and Brémond 2002; Alcocer-Yamanaka, Tzatchkov, and Arreguin-Cortes 2012). Babayan et al. (2005) argued that a network might not deliver dependable service if the design does not consider fluctuations in some or all of the design parameters (e.g. nodal demands, pipe roughness, etc.). Thus, the design problems become more stochastic (Yang et al. 1996; Ostfeld 2004) than deterministic in nature. When demands vary with time, extended period simulations need to be performed to understand the system's dynamic behaviour (Filion and Karney 2002;

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Tabesh, Tanyimboh, and Burrows 2004). Once a stochastic model of daily water demand is presented (Alvisi, Franchini, and Marinelli 2003; Alcocer-Yamanaka, Tzatchkov, and Buchberger 2008), a multi-objective approach (Giustolisi, Laucelli, and Colombo 2009) is then required for designing the WDN considering both nodal demands and pipe diameters as uncertain variables. A technique for stochastic analysis of WDN was developed by Nel (1993) and was then improved by Haarhoff and Van Zyl (2002). The method was further improved by incorporating an extended period analysis technique in EPANET (Rossman 2000) hydraulic engine. However, the computation time increases nonlinearly when the same network is solved repeatedly for stochastic demands, control and operation of various components and optimal design (Kapelan, Savic, and Walters 2005; Giustolisi, Laucelli, and Colombo 2009). It was reported (Deuerlein 2008; Broad, Maier, and Dandy 2010; Martínez-Alzamora, Ulanicki, and Salomons 2014) that the requirement of sufficient computation time was hindering the progress in developing optimization methods for a WDN. In this context, an efficient network reduction method can significantly reduce the overall computation time without compromising the network system's actual hydraulics.

The network reduction methods can help in reducing the size of the prototype network by preserving the nonlinearity of the original network and approximating the operation accurately under various conditions. Jung, Boulos, and Wood (2007) derived formulae for calculating equivalent diameter for series and parallel pipes based on the Hazen-Williams head loss equation. To overcome the limitations in Jung, Boulos, and Wood (2007), a parameter-fitting approach was proposed (Anderson and Al-Jamal 1995) using nonlinear programming. This approach generates multiple solutions – a simplified network can be arranged in many ways, even with only four or five nodes. The hydraulic solutions for the sub-network thus depend on the choice of the topology of the simplified network. As a result, the solution is found to be not unique. Later, several network simplification methods were proposed, such as skeletonization (Walski et al. 2003; Saldarriaga et al. 2008), decomposition (Deuerlein 2008), ANN-based metamodels (Rao and Alvarruiz 2007; Broad, Maier, and Dandy 2010), and variables elimination (Ulanicki, Zehnpfund, and Martinez 1996). The application of skeletonization is limited because the method is focused only on pipe removal and hence suitable only for looped pipe networks. In recent years, ANN is used in WDNs for pressure management, leakage analysis, and water quality modeling (Mosetlhe et al. 2018). The ANN-based network models (Rao and Alvarruiz 2007; Broad, Maier, and Dandy 2010) are not suitable for network analysis, especially for network optimization due to time-demanding training processes. In addition, a small change in the network demands repetition of the entire training process. Clustering is another network reduction process, which partitions a set of objects into subsets of similar properties. The simplified system graph through clustering provides better visualization, relatively easy analysis, and application of appropriate solution techniques (Perelman and Ostfeld 2011). Several graph-theoretic techniques were used recently to speed up the decomposition process. In the reformulated co-trees method (RCTM) (Elhay et al. 2014), the incidence matrix is manipulated into trapezoidal form, in which the lower triangular block at the top represents a spanning tree and the rectangular block below it represents the corresponding co-tree. Whereas, in the forest-core partitioning algorithm (FCPA) (Simpson, Elhay, and Alexander 2014), the linear treed part of the network (the forest) from the nonlinear looped part (the core) is separated by inspecting the incidence matrix. The linear and nonlinear parts of the problem are solved separately by linear and nonlinear methods, respectively. Qiu et al. (2019) compared the performance of the above two methods with the most widely used global gradient algorithm (GGA) (Todini and Pilati 1988), which analyses a WDN using the nonlinear system of equations. However, this method involves several steps in the decomposition of the network to obtain a block graph and the re-modelling of the existing network cannot be avoided. Martínez-Alzamora, Ulanicki, and Salomons (2014) proposed a reduction method for large networks by linearizing the nonlinear model at an operating point and eliminating variables at some nodes from the linear system. An approximate nonlinear model is then recovered from the reduced linear model for computing the hydraulic variables. In this method, the demand at an eliminated node is redistributed among other nodes connected to the eliminated node, which may introduce some error. It is also impossible to eliminate certain non-demand junctions, which are connected to tanks, reservoirs, and pumps. In addition, this method requires several iterations to derive the reduced network and a separate matrix reduction algorithm to efficiently use the Gaussian elimination procedure.

All the network reduction methods described above consider the entire WDN even when an existing WDN is expanded by adding a sub-network with the size of 20–30% of the existing WDN. Ideally, the existing WDN should not add unnecessary computational burden while the focus is primarily on the optimal design of the sub-network. Under this circumstance, a network reduction method, which reduces the existing WDN without its operation may be useful for focused analysis of the sub-network. Such a network reduction method is quite popular in electrical circuits. However, the similarity between an electrical circuit and a pipe network has not been exploited so far. Circuit theories are frequently used in biology for modeling and analysing problems, such as a heart pump model based on a phenomenological characterization of hemodynamics using an electrical analog circuit (Mossa 2008). Oh et al. (2012) reviewed the application of electric circuit methods for the analysis of pressure-driven microfluidic networks with an emphasis on concentration- and flow-dependent systems. Liu and Hodges (2014) developed a software tool called Simulation Program for River Networks (SPRINT) using microprocessor analysis methods for studying Continental River Dynamics.

Only a few preliminary but notable studies are available on the application of circuit theories to analyse pipe network flow (McIlroy 1950; Stephenson and Eaton 1954). Thevenin published a series of studies on electric networks (Thevenin 1883a, 1883b, 1883c, 1883d, 1883e). The studies prove that any linear two-terminal network can be replaced with its equivalent having a single source and a single resistor. Today, Thevenin theorem (Johnson 2003) serves as a handy tool for the circuit designers to perform a focused analysis on a particular portion of a circuit by replacing the whole complicated network with a simple equivalent circuit consisting of only a single source connected with an equivalent resistance (or impedance if reactive elements are present) (Alexander and Sadiku 2000). Hence, the Thevenin theorem can potentially be used to reduce a large, complicated pipe network with a source (i.e. a reservoir) and a resistor (i.e. an equivalent pipe friction) for focused analysis of a particular segment of a large WDN, for example, a sub-network for an expanded area.

In this study, the authors, for the first time, demonstrate the use of the Thevenin theorem to simplify sizable hydraulic pipe networks. The theorem is used to reduce a large pipe network to a simple system consisting of an equivalent pipe connected in series with a single supply source, i.e. a reservoir, and thus achieves significant computational efficiency. The adoption of the theory from an electrical circuit to a hydraulic pipe network is explained in detail. In the absence of an equivalent tool in the hydraulic domain, an electrical circuit simulator called LTspice (Brocard 2013) is also used for finding the Thevenin equivalent voltage and equivalent resistance of the analogous linear electrical circuit corresponding to the original pipe network. The equivalent pipe with an equivalent reservoir head represents the reduced network of the existing WDN. The reduced network is then attached to a sub-network planned for expansion through a single pipe, and the integral system is solved in EPANET (Rossman 2000) for various demand scenarios. The stochastic demands in the sub-network are accounted for through extended period simulations. The results obtained from the equivalent system are compared with the solutions of the original network. Inspired by the adoption of the Thevenin theorem to hydraulic networks, a simplified approach is also presented using the system curve so that one can avoid extra steps for obtaining the Thevenin equivalent network, albeit compromising the accuracy to some extent. It is expected that the proposed network reduction method can be a rapid assessment tool for hydraulic engineers.

Methodology

Concept of network equivalence

According to the electrical circuit theory (Van Valkenburg 1974), two networks are said to be equivalent if they produce the same sets of node potentials and branch currents in an arbitrary network when connected independently. Here, the authors attempt to elaborate on this idea of equivalence in terms of pipe networks using Figure 1. Let A and B represent two different networks connected independently to an arbitrary test network C having *p* number of nodes and *m* number of pipes (Figure 1). Assume that Q_{iA}; i= 1, 2, . . ., m are the resulting flows through the pipes and H_{kA}; k= 1, 2, . . ., p are the heads at the nodes in the arbitrary test network C, when it is connected to the network A, and similarly, Q_{iB} and H_{kB} are the flows and heads in the network C, when it is connected to the network B, respectively. Then network A is said to be equivalent to network B, or these two networks are interchangeable, if and only if $Q_{iA} = Q_{iB}$ and $H_{kA} = H_{kB}$ for all i and k. It should be noted that though these equivalent networks (A and B) produce the same sets of heads and flows in the test network C, the networks A and B can have different topologies. Therefore, any network can have multiple equivalent networks.

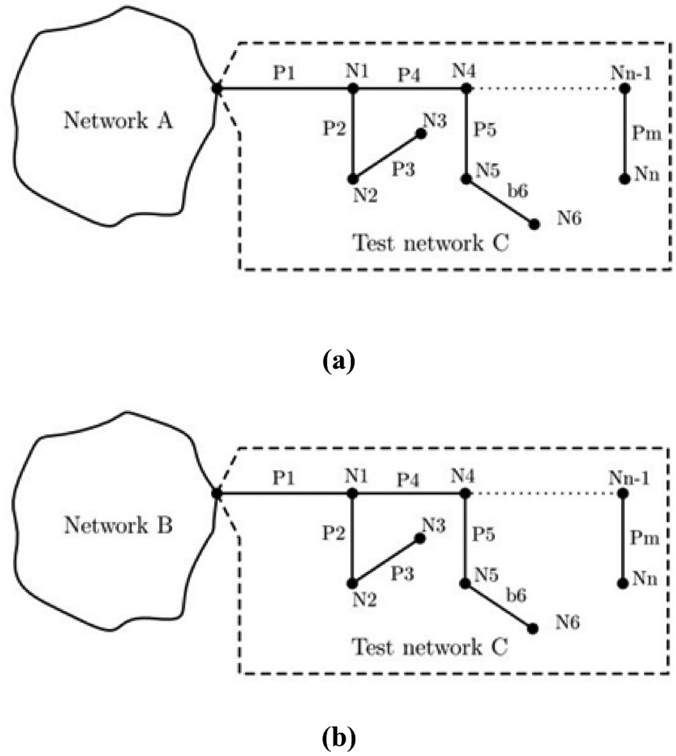


Figure 1. Elaboration of the concept of equivalence: (a) network A connected with arbitrary test network C; and (b) network B connected with arbitrary test network C.

Thevenin theorem and reduction of a pipe network

One can replace an arbitrarily complex but linear two-terminal electrical network using the Thevenin theorem (Alexander and Sadiku 2000) with an equivalent network consisting of a voltage source V_{Th} (Thevenin voltage source) in series with a resistor R_{Th} (Thevenin resistance). V_{Th} is equal to the open-circuit voltage between the two terminals, and R_{Th} is the equivalent resistance when all the independent sources in the network are turned off. R_{Th} is equal to the ratio of V_{Th} and short-circuit current (I_{sc}) (Alexander and Sadiku 2000). V_{Th} is the voltage measured across the open terminal of the circuit, and I_{sc} is the current measured in between the terminals when they are short-circuited. Analogously, a complex pipe network with any number of hydraulic elements can be replaced by an equivalent source with the reservoir head H_{eq} connected in series with an equivalent pipe of resistance K_{eq} using the Thevenin theorem. The H_{eq} is nothing but the piezometric head at the connecting node between the two networks when the demand is made zero at that particular node. Similarly, K_{eq} of the hydraulic network with respect to the connecting node can be obtained by keeping all independent sources like reservoirs, pumps, etc. and all nodal demands in the network inactive. In order to find K_{eq}, one needs to estimate the maximum possible flow (Q_{sc}) through the earmarked node (N_c) when it is open to the atmosphere, which is analogous to the short-circuit current I_{sc} in the electrical circuit.

However, the application of the Thevenin theorem to the pipe network system is not straightforward since the hydraulic network is not a linear system. Unlike in the electrical circuits, where a linear relationship $R = V/I$ (Alexander and Sadiku 2000) is valid, the hydraulic network follows a nonlinear relationship $K = H/Q^n$, in which n depends on the adopted head loss formula with $n > 1$. Therefore, the application of the Thevenin theorem to hydraulic network systems demands linearization of the network with any existing linearization methods (e.g. Martínez-Alzamora, Ulanicki, and Salomons 2014). The proposed methodology is broadly divided into four steps as discussed below.

Step-1: Inspection of the main network to estimate the maximum allowable demand through a node earmarked as the connecting node for network expansion.

Step-2: Linearization of the main network and application of the Thevenin theorem for network simplification.

Step-3: Calculation of parameters for the Thevenin equivalent hydraulic network.

Step-4: Replacement of the existing main network by the Thevenin equivalent hydraulic network and testing the sub-network for various demand patterns.

As discussed, a large pipe network can be reduced to a simple equivalent network consisting of a single reservoir with a head of H_{eq} and an equivalent pipe with a resistance of K_{eq} provided one can apply the Thevenin theorem appropriately. Once this is done, the size of the hydraulic network is reduced significantly. A flowchart is provided in Figure 2 in order to explain the workflow of network simplification by the Thevenin theorem. A detailed description of the above four-steps is provided below.

Step-1: Inspection of the main network to estimate the maximum allowable demand through a node earmarked as the connecting node for a network expansion

The connecting node in the main network is first identified for expansion of the existing network to serve a new area through a sub-network. In general, the node near the sub-network and having a surplus head is selected as the connecting node. This choice does not limit the proposed WDN expansion strategy since this node can be variable and similar analysis can be performed for other nodes. The main network is simulated under varying demands (Q_{cn}) through the chosen connecting node, and the head values at all the nodes of the main network are computed. Nodal head values are expected to reduce with increasing Q_{cn}. In case any of the nodal heads reaches the minimum allowable head value (H_{min}), the demand through the connecting node is noted and marked as the maximum possible demand (Q_{max}) for a sub-network to be connected through that connecting node. In the same way, a constraint on supply, pump capacities, and flow velocity through the pipes of the main network can be set for estimating the maximum allowable demand that can be met through the connecting node. Note that such a rigorous study needs to be carried out only once for each possible node that can be a potential connecting node between the main and the sub-networks. However, it is expected that WDN expansion in a particular area happens through a particular node of an existing main network. If the total demand of the upcoming sub-network is less than or equal to Q_{max}, it will be possible to expand the main network through the connecting node without affecting the allowable head at any node (or flow through any pipe or both heads and flows) in the main network. Thus, Q_{max} ensures that the operation of the main network will not be affected due to expansion, and the main network needs no major reorganization for its hydraulic elements. Once this condition is met, one can go to step-2.

Step-2: Linearization of the main network and application of Thevenin theorem for network simplification

Prior to the application of the Thevenin theorem to an existing WDN, it has to undergo a linearization process. The linearization can be achieved using any standard approach (e.g. Martínez-Alzamora, Ulanicki, and Salomons 2014). In this study, a simple linearization method is adopted to demonstrate the proposed network simplification method. The main WDN is implemented in EPANET and simulated at a fixed operating point Q_{op}. If the main network is linearized at $Q_{op} = Q_{max}/2$, the sub-network can be analysed for a wide range of demand variations without compromising the overall accuracy due to linearization. In this process, all the nodal heads H_i and pipe flows Q_{ij} (between the nodes i and j) in the main network are noted for the operating demand $Q_{max}/2$ through the connecting node. The equivalent linear resistance of each pipe is obtained as $R_{ij} = (H_i - H_j)/Q_{ij}$. However, the linearization of the main network can also be done for $Q_{op} = Q_{ds}$, in which Q_{ds} is the estimated demand in the sub-network. Similar to the EPANET source file, an electrical circuit source file is generated using these equivalent resistors following the analogy between electrical and hydraulic networks (McIlroy 1950) to obtain the open-circuit potential V_{Th} and short-circuit current I_{sc} flowing through the connecting node and the ground. For this purpose, an electrical circuit simulator LTspice (Brocard 2013) is used. It should be noted that LTspice is required at this stage because such a tool is not available in the domain of hydraulics. The future implementation of a similar tool in a hydraulic simulator can widen the scope of the application of circuit theories to solve hydraulic problems.

Step-3: Calculation of the parameters for the Thevenin equivalent hydraulic network

The parameters for the Thevenin equivalent hydraulic network are obtained in this step. Note that already V_{Th} and I_{sc} are available in step-2. Again, by using the analogy $H_{eq} = V_{Th}$, K_{eq} of the equivalent pipe is calculated as $K_{eq} = V_{Th}/[I_{sc} \times (Q_{op})^{n-1}]$. The estimation of equivalent head of the reservoir and equivalent resistance of the main network for network reduction is the important contribution of this study. One can now easily calculate the diameter or length of the equivalent pipe of the Thevenin equivalent hydraulic network using the chosen head loss formula. For example, the length of the equivalent pipe can be calculated as $L_{eq} = (K_{eq}\, C_{HW}{}^{1.852}\, D^{4.87})/10.67$ by assuming the value of the Hazen-William's coefficient C_{HW} for a given pipe diameter D (Larock, Jeppson, and Watters 1999).

Step-4: Replacement of the existing main network by the Thevenin equivalent hydraulic network and testing the sub-network for various demand patterns

Finally, the sub-network is connected with the Thevenin equivalent hydraulic network at the connecting node. The sub-network connected with the Thevenin network is now simulated in EPANET for various sub-network demand patterns. The sub-

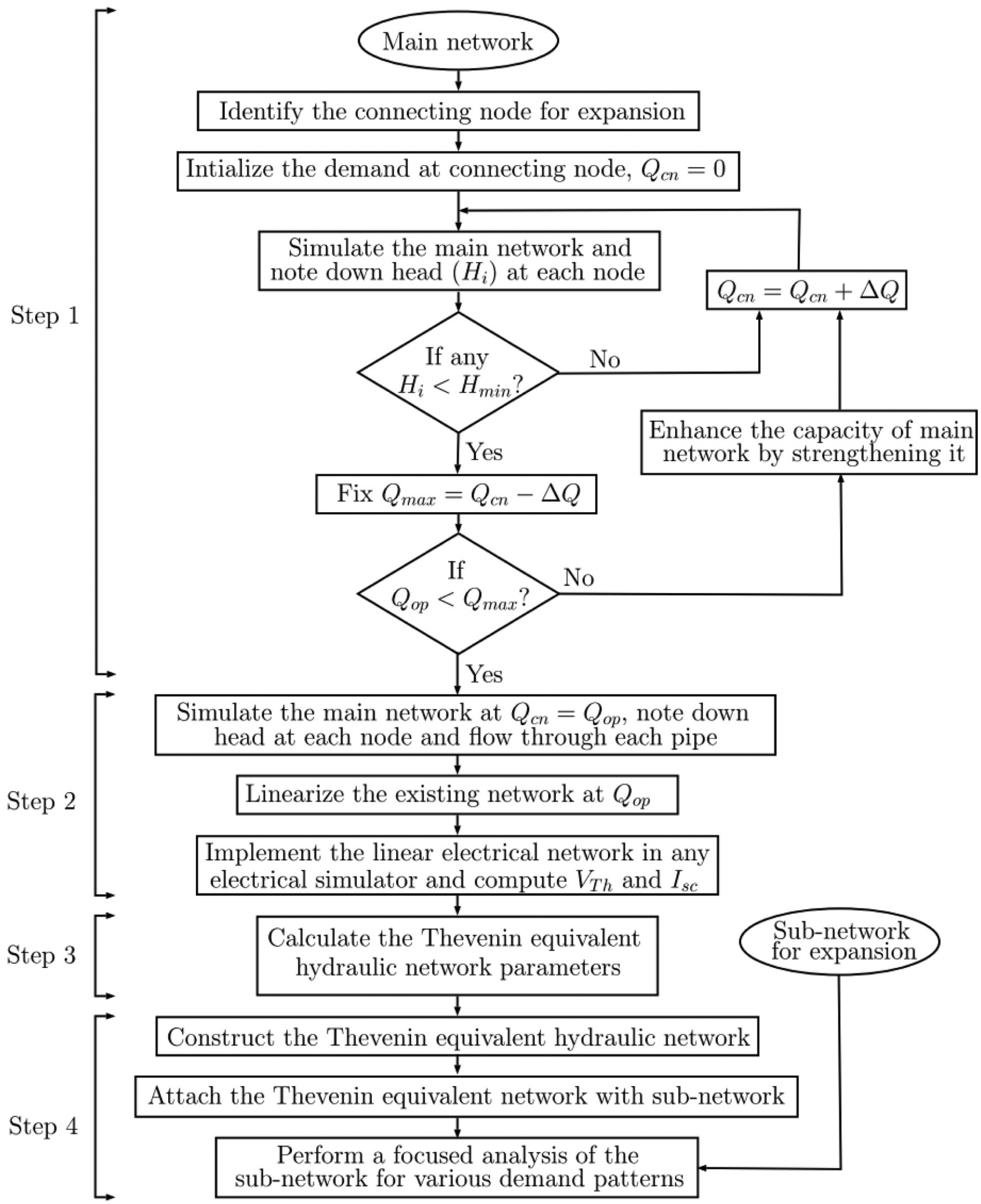


Figure 2. Flow chart for the proposed methodology.

network can also be tested for various pipe configurations or layouts as long as the total demand in the sub-network is within Q_{max}.

Results and discussion

The proposed network reduction method using the Thevenin theorem is examined by solving a small and a relatively large WDN when connected with a sub-network to analyse network expansion problem. It is important to note here that the same concept can be applied to analyse a large existing network for changes in demand, layout, pipe configuration, etc. in a particular portion, i.e. a sub-network. In the following applications, the WDNs are first tested for different demands using steady state solution method. Second, the WDNs are again tested using extended period simulations. The results of the reduced networks are compared with the solutions of the full networks. It may appear that the implementation of the proposed methodology is not straightforward due to the use of hydraulic and circuit simulators. Under this circumstance, a simplified approach derived from understanding the Thevenin theorem can help hydraulic engineers solve similar problems, but the associated error needs to be examined. For this purpose, a system curve at the connecting node is prepared by varying demands at the connecting node and observing the corresponding heads. In the first step, the head H_{nd} at the connecting node is obtained by simulating only the main network for zero sub-network demand. This calculation of head loss is originated from the idea of Thevenin equivalent head, which is H_{nd} in this case. Then, head loss H_L in the main network for a total sub-network demand at a given instant is calculated by subtracting the computed head for that sub-network demand from the head H_{nd} calculated in the previous step. The system resistance is now calculated using the formula $K = H_L/Q_{ds}{}^n$. For the extended period simulations, an equivalent system resistance (K_{eq}) is calculated for the entire simulation period. If twenty-four different demands (each for every hour interval) are specified for a full day, we obtain twenty-four different K values. Subsequently, using the least square fit over the twenty-four different K values, we obtain the equivalent resistance (K_{eq}). If the diameter and Hazen-Williams coefficient of the equivalent pipe are fixed as D_{eq} and C_{HW}, then to satisfy K_{eq}, the length of the equivalent pipe must be L_{eq}. Therefore, an equivalent network for the main network can thus be obtained by setting H_{eq} = H_{nd} (inspired by the Thevenin theorem) and using the equivalent parameters as calculated above. The equivalent network can now be connected with the sub-network for focussed analysis. Although the simplified approach presented above can avoid the linearization step outlined in Figure 2, it introduces relatively large errors compared to that obtained from the proposed network reduction method as discussed in the following application. However, such errors may be acceptable for analysis considering the associated uncertainties in the design steps of a WDN. The computation times for all the applications presented below are run for one thousand times and the average values are reported herein for comparison.

Application of the proposed network reduction method to a realistic WDN (step-by-step implementation)

To demonstrate the application of the circuit theory mentioned above to the hydraulic pipe network system, a particular zone of a WDN (Figure 3) in a small city in Southern India is considered here. The details of the network can be found in Kumar, Narasimhan, and Bhallamudi (2008). The hydraulic network consists of 71 pipes, one reservoir, and 45 demand nodes. The length (L), diameter (D) and other details of the pipes are obtained from the cited reference. However, in the present study, the pipe P45 (represented by a thick dashed line in Figure 3) between the nodes N34 and N36 is removed to demonstrate the application of the single-port Thevenin theorem. The network is divided into main (dashed part) and sub-network (dotted part) as marked in Figure 3. The main upstream network has 63 pipes, one reservoir, and 40 demand nodes and the sub-network has 7 pipes and 5 demand nodes (Figure 3). After replacing the main network with its Thevenin equivalent hydraulic network, the original network shown in Figure 3 is reduced as shown in Figure 4. The freely available pipe flow simulator EPANET version 2.0 (Rossman 2000) and the circuit simulator LTspice (Brocard 2013) are used, respectively, for the hydraulic and circuit simulations.

Step-1: Inspection of the main network to estimate the maximum allowable demand through a node earmarked as the connecting node for network expansion

The node N15 of the main network is identified as the connecting node between the main and sub-networks. The demand at this node is varied from 0 m^3/s to 28×10^{-3} m^3/s with the step size of 4×10^{-3} m^3/s. For all these demands, the heads at all the nodes of the main network are compared with the minimum required head; for example, in this case H_{min} is set at 7 m to serve single-storey buildings (CPHEEO 1999). From Figure 5(a), it is evident that N14 is first going below 7 m. So, this is the most sensitive node in the main network for variations in the sub-network demand. For finding the maximum possible sub-network demand (Q_{max}), the variation in head at N14 for increasing connecting node demand and H_{min} line are plotted in Figure 5(b). The demand at which the H_{min} line touches the head variation curve is marked as the Q_{max} and is found to be 26×10^{-3} m^3/s. Therefore, the main network can meet an extra demand of 26×10^{-3} m^3/s through the connecting node without affecting its operation.

Step-2: Linearization of the main network and application of the Thevenin theorem for network simplification

The operating demand for the sub-network is calculated as $Q_{op} = Q_{max}/2 = 13 \times 10^{-3}$ m^3/s. As described before, the linearization process is implemented for the main upstream network by disconnecting the sub-network at node N15, where the total demand of the sub-network (Q_{op}) is also assigned. The linear resistance values of the pipes are then calculated as shown in Table 1 from the EPANET simulations of the main network. Subsequently, by using the analogy between the electrical and hydraulic networks, an LTspice netlist is prepared as shown in Figure 6. This netlist is used for finding V_{Th} and I_{sc} at the node N15. The open circuit voltage is obtained by analyzing the circuit after opening the node N15, ensuring a zero-current flow, which is similar to zero sub-network demand. The short circuit current is obtained by simulating the netlist after adding the element R39 (through which the connecting node N15 shorted to the ground). By simulating the netlist appropriately, the open circuit voltage and the short-circuit current are obtained as V_{Th} = 11.67 V and I_{sc} = 0.152 A, respectively.

Step-3: Calculation of the parameters for the Thevenin equivalent hydraulic network

For the operating sub-network demand $Q_{op} = Q_{max}/2$, the Thevenin equivalent reservoir head is calculated as H_{eq} = V_{Th} = 11.67 m. The Thevenin equivalent pipe resistance is calculated as $K_{eq} = 11.67/[0.152\times(0.013)^{0.852}] = 3105.61$. Now, the main network can be replaced with its Thevenin equivalent consisting of a single source reservoir with the equivalent head of 11.67 m connected in series with an equivalent pipe having K_{eq} = 3105.61. For simulations in EPANET, Hazen-William's coefficient C_{HW} = 120 and equivalent diameter D_{eq} = 0.1 m are considered (Kumar, Narasimhan, and Bhallamudi 2008), and L_{eq} = 27.84 m is calculated from the estimated K_{eq} using the same head loss formula.

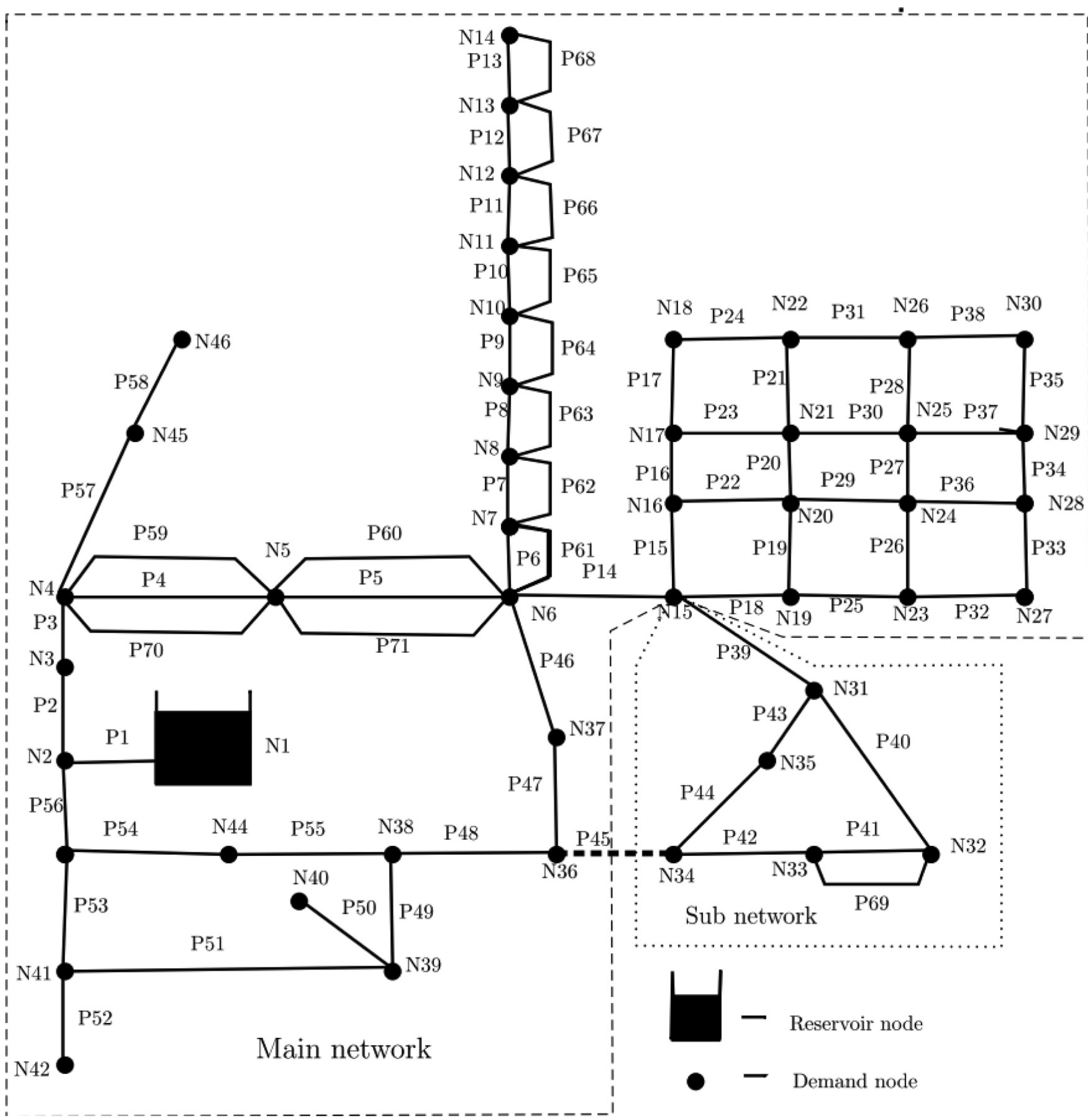


Figure 3. The original pipe networks.

Step-4: Replacement of the main network by the Thevenin equivalent hydraulic network and testing the sub-network for various demand patterns

The Thevenin equivalent network is now connected with the sub-network to analyze the sub-network under various possible demands (Figure 4). The variations in demand can be a set of deterministic demands over a typical period, such as a 24-hour day or daily average or peak demands or highly stochastic in domestic consumption (Alcocer-Yamanaka, Tzatchkov, and Arreguin-Cortes 2012). The consideration of variations in demand is a more realistic approach for the design and reliability analysis of a water supply system. Therefore, the authors want to evaluate both the accuracy and efficiency of the reduced Thevenin pipe network (Figure 4) for various demands by comparing it with the reference solutions. The reference solutions are obtained by solving the original full network (Figure 3) using EPANET.

Evaluation of Thevenin equivalent hydraulic network for steady state simulations

The reduced network and the full network are now simulated in EPANET for all the base demands, as shown in Table 2. The base demand is the average or nominal demand by the main category of consumers at a junction. After simulating both the networks shown in Figures 3 and Figure 4, the discharges in different pipes of the sub-network are compared, and it is found that the flows in the pipes of the sub-network, when it is connected to the Thevenin equivalent network are the same ($|\Delta Q| = 0$ m^3/s) with those, when it is connected to the main upstream network. The resulting percentage errors in the head values of the reduced network are shown in Figure 7. Since the upstream network is linearized at $Q_{op} = Q_{max}/2$, the results of the reduced network exactly match with the full network at this operating point of $Q_{max}/2$. The reduced network yields a high level of accuracy

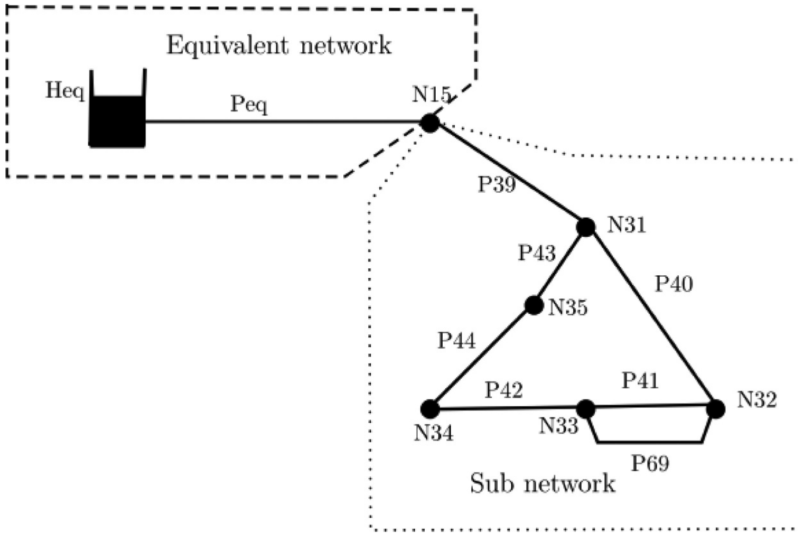


Figure 4. Equivalent pipe network for the original network shown in Figure 3.

with an error less than 2.5% for the other demand factors. The worst-case error is 4.7%, which occurs for a zero demand in the sub-network. These errors are obviously introduced due to the simple linearization of the nonlinear upstream main network for a specified demand (Q_{op}) in the sub-network.

From the above discussions, it is clear that the results obtained from the Thevenin equivalent hydraulic network depends on the demand at which the main network is linearized. If it is required to have zero error, the linearization of the main network needs to be done for the known demand in the sub-network, i.e. $Q_{op} = Q_{ds}$, in which Q_{ds} is the total estimated demand in the sub-network. Otherwise, for example, for the analysis of stochastic demand and reliability, linearization of the main network at $Q_{op} = Q_{max}/2$ is the best option. Overall, it is observed that the results from the reduced network are in excellent agreement with the reference solutions.

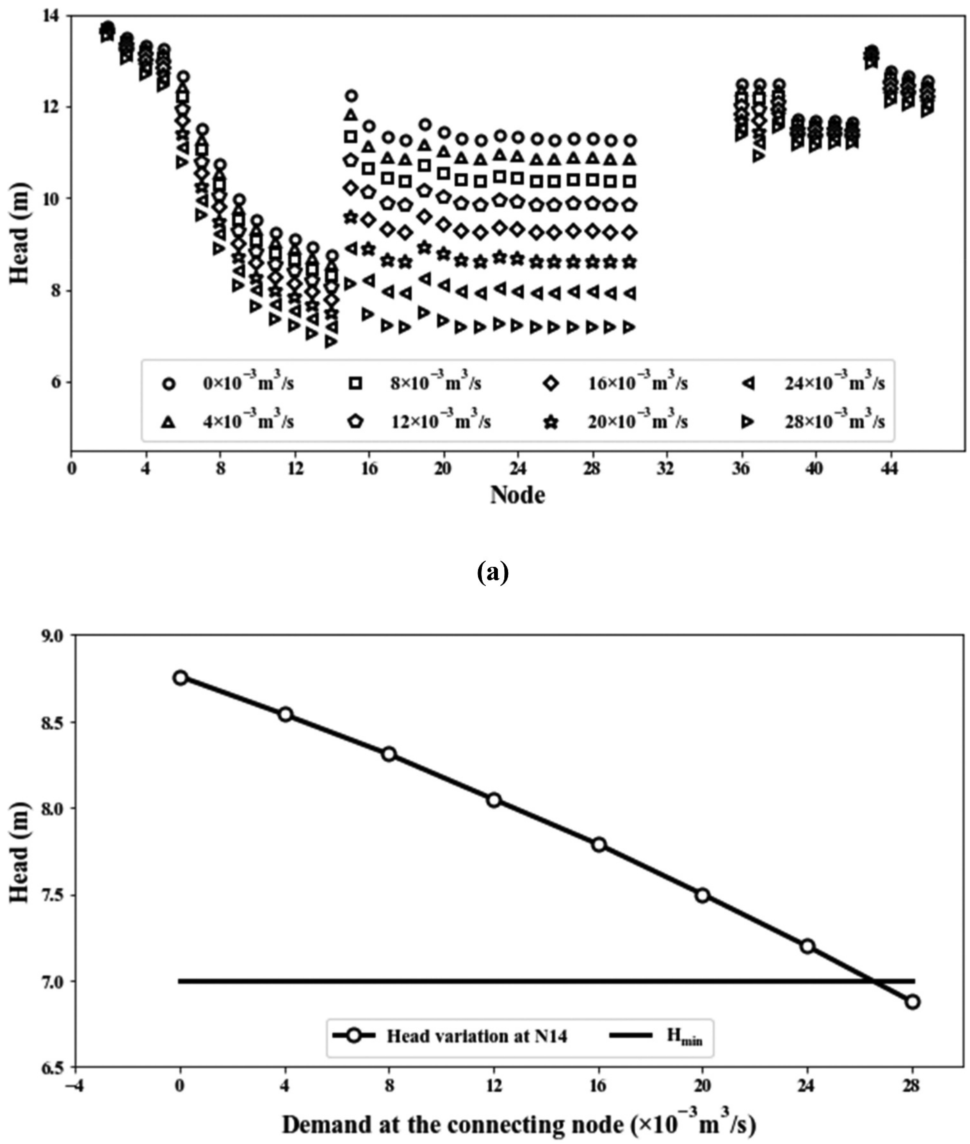
The computation times for these two network configurations are also investigated. It is observed that the network configuration in Figure 4 can be simulated two times faster than the network configuration in Figure 3 using EPANET. Such a reduction in computation time can be useful for focused analysis of a sub-network under various constraints such as stochastic demands, different layouts, pipe sizes, optimal design, etc. as long as the total demand in the sub-network is within Q_{max} and the main network is linearized at Q_{op}.

Evaluation of Thevenin equivalent hydraulic network for extended period simulations

In case of WDN analysis, it is important to predict how a system will behave when subjected to different operating states, such as variations in demand over a typical period. A set of deterministic and stochastic demands over a typical period of 6, 12, 24, or 48 hours are considered to assess the adequacy of pressures, flows, and velocities in the WDN through extended period or dynamic simulations (Filion and Karney 2002). Moreover, dynamic models are often used to make key planning, design and operation decisions such as sizing of pipes, reservoirs, and pumps in a system, timing of operation of pumps and developing control strategies to maintain an acceptable level of service (Rao and Bree 1977; Bhave 1991).

In this application, the same networks in Figures 3 and Figure 4 are considered for extended period simulations. Three demand patterns are generated with the demand factors as given in Table 3 for a time period of 24 hours. The hydraulic time step is considered as 1 hour and the reporting time step is fixed at 15 minutes. To evaluate the reduced hydraulic network, five different cases (Table 2) are considered for extended period simulations. In each case the Pattern 3 is assigned to all the main network nodes as base demands. The base demands and patterns at each sub-network node for different cases are given in Table 2.

Both the reduced and the main networks are found to be giving the same flow patterns with the same magnitudes (i.e. $|\Delta Q| = 0$ m^3/s) in the sub-network for all the considered cases. The sub-network node N34 is found to have the maximum variation in head values for the changes in the sub-network demands. Therefore, N34 is chosen for further comparisons. Figure 8 compares the head values at N34 of the sub-network at different time periods for all the cases. From this figure it is evident that both the reduced and the main networks are producing the same head patterns with the same magnitudes at the operating demand of 13 lps. When the sub-network demand is other than the operating demand, both the networks produce the same head patterns but with small differences in magnitudes. The maximum and average percentage errors in head values at different sub-network nodes for different demand patterns are given in Table 4. This Table 4 shows that the maximum error for all the cases at different nodes is less than 5%, and the average error is less than 3%. Therefore, it can be concluded that the proposed network reduction method can produce similar level of accuracy even for extended period simulations, which can help in analyzing stochastic demands and reliability of a network. The computation times for these two network configurations are also investigated. The proposed network reduction method is observed to be 1.5 times faster than the traditional extended period simulation of the full network.



(a)

(b)

Figure 5. Finding the maximum possible sub-network demand: (a) heads at all nodes of main network for various connecting node demands; and (b) heads at N14 of main network for various demands at the connecting node.

Comparison of results obtained using the Thevenin equivalent hydraulic network and the system curve approach

In the system curve approach, the total sub-network demands for Case 1 (Table 2) at different time intervals are calculated as given in Table 5. The head values corresponding to the sub-network demands are measured and noted in Table 5 using the system curve shown in Figure 9. The head at the connecting node for zero sub-network demand is computed as H_{nd} = 12.25 m. The system resistance at each time interval is now calculated using the formula $K = H_L/Q_{ds}^n$. Applying the least square technique over the twenty-four different K values obtained from the hourly demands in a day, the equivalent system resistance for the entire simulation period is estimated as K_{eq} = 6263.5. If the diameter and Hazen-Williams coefficients of the equivalent pipe are fixed as D_{eq} = 0.1 m and C_{HW} = 120, then to satisfy K_{eq}, the length of the equivalent pipe must be L_{eq} = 56.92 m. The above calculations are required beforehand to use the EPANET simulator.

To test this simplified methodology, the main network is replaced with its equivalent consisting of a reservoir with the head H_{eq} = H_{nd} = 12.25 m and an equivalent pipe having parameters obtained from the system curve. The base demands and patterns given in Table 2 are assigned in the sub-network nodes for extended period simulations. The corresponding heads at N34 are noted in Table 6. In the same table, the results obtained from this method are compared with the reference solutions and the results obtained using the proposed

```
; Reservoir - DC voltage source
; ID      Node   Ref. Node   Value (V)
  V       N1     0           20
; Demand - DC current source
; ID      Node   Ref. Node   Value (A)
I2        N2     0           0.00133
I3        N3     0           0.00373
I4        N4     0           0.00385
I5        N5     0           0.00316
I6        N6     0           0.00107
I7        N7     0           0.00189
I8        N8     0           0.00133
I9        N9     0           0.00215
I10       N10    0           0.00278
I11       N11    0           0.00177
I12       N12    0           0.00133
I13       N13    0           0.0012
I14       N14    0           0.00253
I15       N15    0           0.00284
I16       N16    0           0.00126
I17       N17    0           0.00101
I18       N18    0           0.00082
I19       N19    0           0.000505
I20       N20    0           0.000758
I21       N21    0           0.000632
I22       N22    0           0.000378
I23       N23    0           0.000883
I24       N24    0           0.00126
I25       N25    0           0.00107
I26       N26    0           0.000632
I27       N27    0           0.00297
I28       N28    0           0.000315
I29       N29    0           0.000442
I30       N30    0           0.000378
I36       N36    0           0.00101
I37       N37    0           0.000883
I38       N38    0           0.000568
I39       N39    0           0.00164
I40       N40    0           0.000632
I41       N41    0           0.0124
I42       N42    0           0.00208
I43       N43    0           0.00493
I44       N44    0           0.00019
I45       N45    0           0.00619
I46       N46    0           0.00366
; Pipe - linear resistance
; ID    Node1  Node2     value (ohm)
R1      N1     N2        3.72
R2      N2     N3        5.04
R3      N3     N4        3.52
R4      N4     N5        5.84
R5      N5     N6        38.87
R6      N6     N7        153.54
R7      N7     N8        115.01
R8      N8     N9        132.48
R9      N9     N10       90.95
R10     N10    N11       86.38
R11     N11    N12       50.99
R12     N12    N13       106.17
R13     N13    N14       211.46
R14     N6     N15       38.47
R15     N15    N16       127.38
R16     N16    N17       110.89
R17     N17    N18       51.83
R18     N15    N19       80.26
R19     N19    N20       64.79
R20     N20    N21       85.51
R21     N21    N22       44.01
R22     N16    N20       65.79
R23     N17    N21       21.15
R24     N18    N22       2.78
R25     N19    N23       46
R26     N23    N24       30.56
R27     N24    N25       53.07
R28     N25    N26       24.79
R29     N20    N24       57.95
R30     N21    N25       25.11
R31     N22    N26       10.33
R32     N23    N27       28.49
R33     N27    N28       5.85
R34     N28    N29       18.94
R35     N29    N30       26.37
R36     N24    N28       39.8
R37     N25    N29       10.37
R38     N26    N30       1.44
R46     N6     N37       34.01
R47     N37    N36       67.92
R48     N36    N38       37.52
R49     N38    N39       249.60
R50     N39    N40       47.47
R51     N39    N41       58.17
R52     N41    N42       14.42
R53     N41    N43       112.52
R54     N43    N44       124.25
R55     N44    N38       84.58
R56     N43    N2        22.05
R57     N4     N45       69.04
R58     N45    N46       32.79
R59     N4     N5        18.13
R60     N5     N6        120.67
R61     N6     N7        153.53
R62     N7     N8        117.25
R63     N8     N9        132.48
R64     N9     N10       90.94
R65     N10    N11       86.38
R66     N11    N12       50.99
R67     N12    N13       106.17
R69     N32    N33       93.47
R70     N4     N5        12.44
R71     N5     N6        82.83
;for calculating V oc, R39 is disconnected from the circuit
R39     N15    0         0.0001
.op  ;analyzes the circuit for given operating conditions
```

Figure 6. LTspice netlist for finding V_{oc} and I_{sc} at node N15 of analogues linear electrical network of the main network shown in Figure 3.

Thevenin equivalent network. From this comparison, it is evident that the Thevenin equivalent network provides more accurate results than that obtained from the system curve method. The difference in accuracy originates from the equivalent reservoir heads of the reduced networks. The equivalent head for the system curve approach is different because this is the head at the connecting node for zero sub-network demand obtained without linearizing the main network. It needs to be stressed here that the Thevenin method is more generalized and the parameters depend only on the demand at which the existing network is linearized. Although the system curve method can avoid the linearization step involved in the Thevenin network reduction method, the parameter K requires calculation each time with the change in demand pattern even within a given range. On the other hand, in case of the proposed Thevenin equivalent network, the linearization and parameter calculation processes need not be repeated as long as the main network remains unaltered.

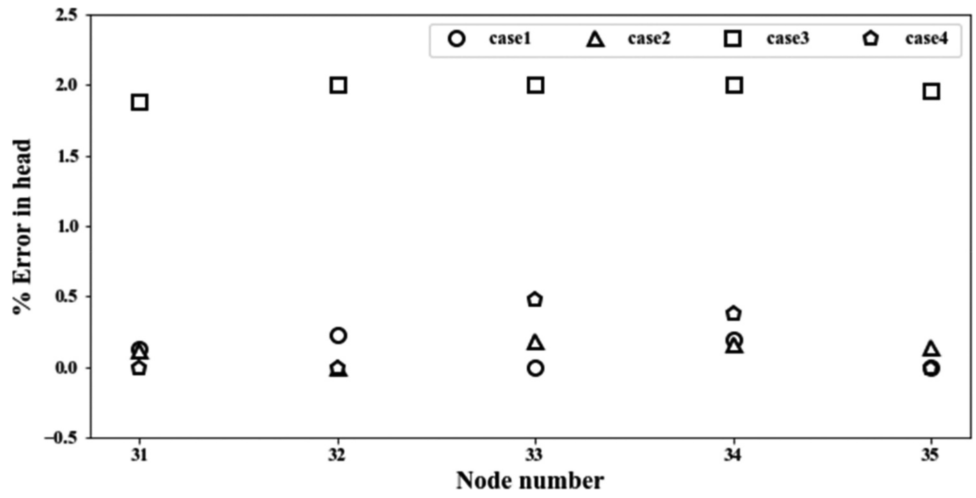


Figure 7. Evaluation of Thevenin equivalent hydraulic network: percentage error in heads at different nodes as obtained from the Thevenin network in comparison with the original network.

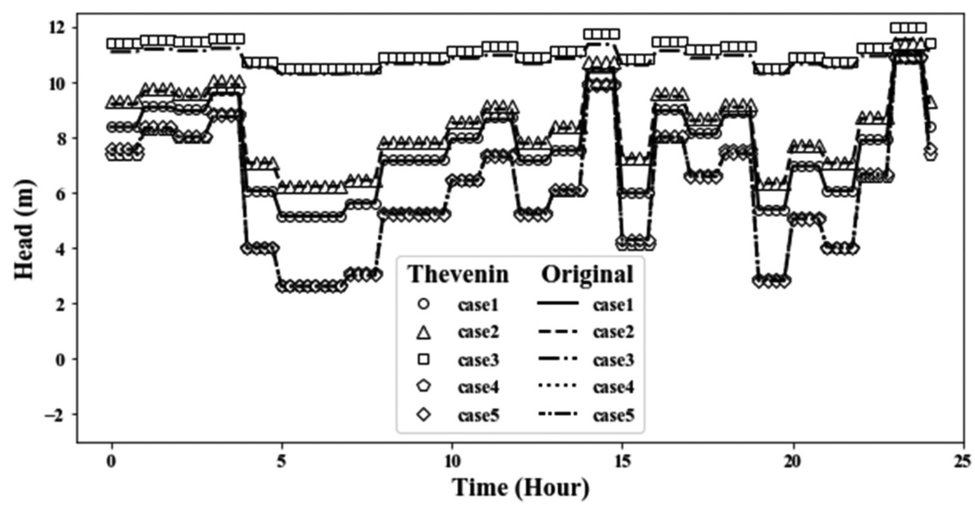


Figure 8. Comparison of head values at different sub-network nodes for different demand patterns in Table 3.

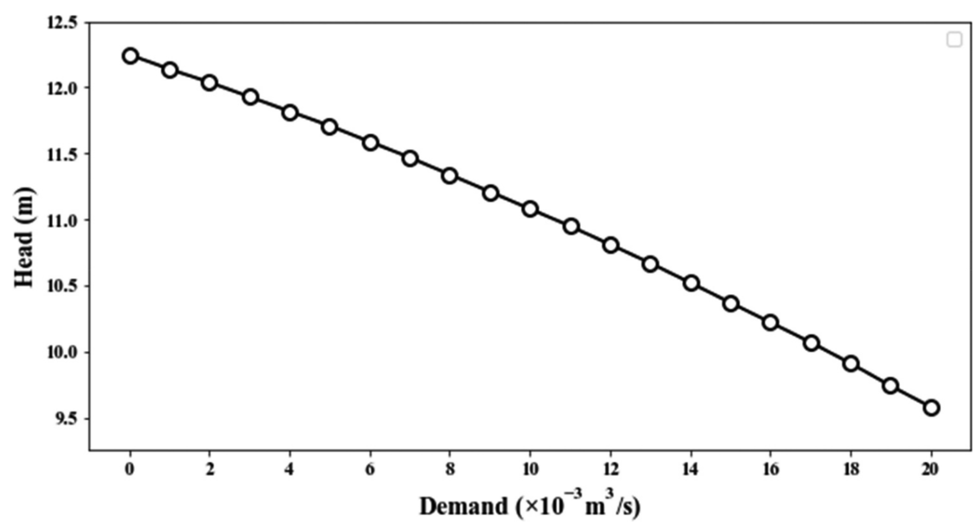


Figure 9. System head curve at connecting node (N15).

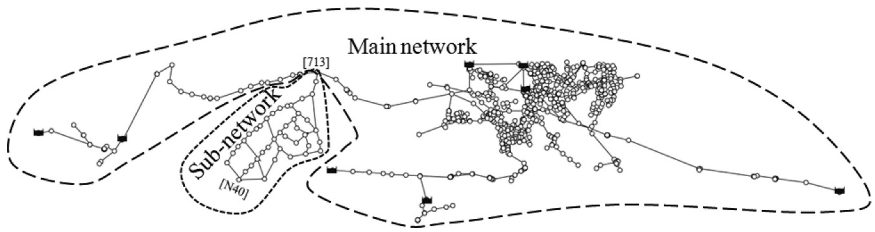


Figure 10. Richmond water distribution network (dashed part) with a hypothetical sub-network (dotted part).

Table 1. Calculation of linear resistances for the pipes in the main network.

Pipe Number	Head loss (m)	Discharge (m^3/s)	linear R (Ω)	Pipe Number	Head loss (m)	Discharge (m^3/s)	linear R (Ω)
1	0.350	0.0915	3.83	33	0.001	0.0002	6.25
2	0.340	0.0658	5.16	34	0.010	0.0005	18.87
3	0.230	0.0621	3.70	35	0.010	0.0004	28.57
4	0.160	0.0270	5.92	36	0.030	0.0010	30.00
5	1.030	0.0253	40.79	37	0.001	0.0003	3.70
6	1.150	0.0075	153.54	38	0.001	0.0000	33.33
7	0.750	0.0066	114.50	46	0.001	0.0000	33.33
8	0.780	0.0059	132.65	47	0.080	0.0009	94.12
9	0.440	0.0048	91.67	48	0.080	0.0019	43.01
10	0.300	0.0034	87.98	49	0.570	0.0024	242.55
11	0.130	0.0025	51.38	50	0.030	0.0006	47.62
12	0.200	0.0019	107.53	51	0.001	0.0001	12.50
13	0.150	0.0007	205.48	52	0.030	0.0021	14.42
14	1.220	0.0292	41.85	53	1.630	0.0144	113.19
15	0.680	0.0053	127.34	54	0.640	0.0050	128.77
16	0.250	0.0023	111.11	55	0.420	0.0048	87.87
17	0.040	0.0008	51.95	56	0.540	0.0243	22.22
18	0.640	0.0080	80.30	57	0.680	0.0099	69.04
19	0.160	0.0025	64.78	58	0.120	0.0037	32.79
20	0.140	0.0016	85.37	59	0.160	0.0087	18.39
21	0.030	0.0007	44.12	60	1.030	0.0081	126.69
22	0.120	0.0018	65.93	61	1.150	0.0075	153.54
23	0.010	0.0005	21.28	62	0.750	0.0066	114.50
24	0.001	0.0001	20.00	63	0.780	0.0059	132.65
25	0.230	0.0050	46.00	64	0.440	0.0048	91.67
26	0.050	0.0013	38.17	65	0.300	0.0034	87.98
27	0.040	0.0009	42.55	66	0.130	0.0025	51.38
28	0.010	0.0004	25.00	67	0.200	0.0019	107.53
29	0.120	0.0019	63.16	69	0.150	0.0018	83.33
30	0.020	0.0008	25.00	70	0.160	0.0127	12.62
31	0.001	0.0003	3.85	71	1.030	0.0119	86.92
32	0.080	0.0028	28.47				

Application of the proposed network reduction method to a large bench-mark WDN

To evaluate the applicability of the proposed network reduction method on a large WDN, a bench-mark network (Richmond water distribution network) is chosen from the ASCE library (Qiu et al. 2019). This network has 848 nodes, 8 reservoirs and 934 pipes (Figure 10). The pipe and reservoir data, and other details of the network are obtained from the cited reference. The C_{HW} value of all the pipes is fixed to 120. Node elevations are not considered while demonstrating the proposed method. A hypothetical sub-network with 40 nodes and 50 pipes is connected to the main network at node N713 as shown in Figure 10. In the sub-network, length, diameter and C_{HW} of all the pipes are taken as 300 m, 50 mm and 120, respectively for the purpose of demonstration. Also, the demand at each node of the sub-network is assumed to be 0.1×10^{-3} m^3/s.

The main network shown in Figure 10 is now simulated in EPANET considering all the above conditions after removing the sub-network and adding the total demand of the sub-network as $Q_{op} = Q_{d\ s} = 4 \times 10^{-3}$ m^3/s (i.e. 10% of existing network demand) at N713. The resulting heads at each node and flow through each pipe are noted. By using this data, linear resistances of all the pipes in the main network are calculated following the linearization methodology elaborated earlier. A netlist is prepared similar to EPANET input file for calculating V_{Th} and I_{sc} using LTspice. Corresponding values at the connecting node N713 are obtained as $V_{Th} = 236.19$ V and $I_{sc} = 0.258$ A. Finally, the nonlinear resistance of the equivalent pipe is calculated as $K_{eq} = 236.19/[0.258\times(0.004)^{0.852}] = 101085$. Now the main network is replaced with its Thevenin equivalent consisting of a single source reservoir with the equivalent head of $H_{eq} = 236.19$ m connected in series with an equivalent pipe having $K_{eq} = 101085$. For the simulation in EPANET, $C_{HW} = 120$ and $D_{eq} = 0.05$ m are considered for the equivalent pipe, and $L_{eq} = 30.98$ m is calculated from the already obtained K_{eq} value. The Thevenin equivalent network is now connected with the sub-network (Figure 11) for analyzing the sub-network for various possible demand patterns.

Evaluation of Thevenin equivalent hydraulic network for steady state simulations

To examine the accuracy of the Thevenin equivalent hydraulic network shown in Figure 11, both the original and reduced networks are simulated in EPANET for different sub-network demands. The total demand in the sub-network is varied from

Table 2. Base demands (B.D.) (lps) and demand patterns at the sub-network nodes for extended period simulation.

Node	Case 1		Case 2		Case 3		Case 4		Case 5	
No.	B.D. (lps)	Pattern No.	B.D. (lps)	Pattern No.	B.D. (lps)	Pattern No.	B.D. (lps)	Pattern No.	B.D. (lps)	Pattern No.
31	5	1	3	1	1	1	5	1	5	2
32	3	2	3	2	1	2	3	2	3	1
33	3	1	3	1	1	1	3	1	3	2
34	1	2	1	2	1	2	2	2	2	1
35	1	1	1	1	1	1	2	1	2	1

Table 3. Different demand patterns for extended period simulations.

Time	1	2	3	4	5	6	7	8	9	10	11	12
Pattern 1	1	0.8	0.7	0.5	1	1	1	0.8	0.7	0.7	0.7	0.5
Pattern 2	0.2	0.3	0.5	0.6	0.8	1	1	1.2	1	1	0.8	0.9
Pattern 3	1	1	1	1	1	1	1	1	1	1	1	1
Time	13	14	15	16	17	18	19	20	21	22	23	24
Pattern 1	0.7	0.9	0.3	1.2	0.7	0.6	0.4	0.9	0.8	1	0.9	0.3
Pattern 2	1	0.6	0.6	0.5	0.5	0.9	1	1.1	0.9	0.8	0.5	0.3
Pattern 3	1	1	1	1	1	1	1	1	1	1	1	1

Table 4. Maximum and average % errors at different sub-network nodes for different cases.

	Case 1		Case 2		Case 3		Case 4		Case 5	
% Error	Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.	Max.	Avg.
Node 31	2.4	0.5	2.6	0.8	3.7	2.5	2.1	0.3	2.1	0.3
Node 32	2.4	0.5	2.7	0.9	3.8	2.5	2.1	0.3	2.1	0.4
Node 33	2.4	0.5	2.7	0.9	3.7	2.5	2.2	0.4	2.2	0.5
Node 34	2.4	0.5	2.7	0.9	3.8	2.5	2.1	0.4	2.1	0.4
Node 35	2.4	0.5	2.7	0.8	3.8	2.5	2.2	0.3	1.2	0.3

2×10^{-3} m^3/s to 6×10^{-3} m^3/s (i.e. 1.5 times of the operating demand) with a step size of 1×10^{-3} m^3/s. In each case, the total sub-network demand is distributed equally among all the sub-network nodes for the demonstration purpose. The solutions of the full network (i.e. sub-network connected with the original main network) are considered as the reference solutions for evaluating the accuracy of the reduced network. The flow solutions of the reduced network are found to be exactly in agreement ($|\Delta Q| = 0$ m^3/s) with those of the reference solutions. The sub-network node N40 is found to have the maximum variation in head values for the changes in the sub-network demands. Therefore, N40 is chosen for further comparisons of the results. The resulting percentage errors in the head values for the reduced network are shown in Figure 12. Since the upstream network is linearized at $Q_{op} = 4 \times 10^{-3}$ m^3/s, the results of the reduced network exactly match with the original full network at this sub-network demand Q_{op}. The maximum error is found to be less than 1.5% for an extreme value of the sub-network demand, $Q_{ds} = 6 \times 10^{-3}$ m^3/s. Overall, we found that the results from the reduced network are in excellent agreement with the reference solutions.

It is observed that the reduced network shown in Figure 11 simulates four times faster than the original network of Figure 10. Hence, the proposed network reduction method can be very useful for rigorous analysis of a sub-network to be connected with even a large existing WDN.

Table 5. Calculation of system resistance at different time periods using system curve.

Time (Hrs)	Subnetwork demand (Q_{ds}) (lps)	Head at connecting node (m)	Head loss (H_L) (m)	System resistance (K)
1	9.8	11.11	1.14	5986.24
2	8.4	11.29	0.96	6706.66
3	8.3	11.30	0.95	6785.64
4	6.9	11.48	0.77	7743.59
5	12.0	10.78	1.47	5144.92
6	13.0	10.67	1.58	4916.25
7	13.0	10.67	1.58	4916.25
8	12.0	10.81	1.44	5196.59
9	10.3	11.04	1.21	5794.43
10	10.3	11.04	1.21	5794.43
11	9.5	11.15	1.10	6118.55
12	8.1	11.33	0.92	6875.01
13	10.3	11.04	1.21	5794.43
14	10.5	11.01	1.24	5730.32
15	5.1	11.70	0.55	9681.50
16	12.8	10.69	1.56	4995.42
17	8.3	11.30	0.95	6785.64
18	9.0	11.21	1.04	6394.05
19	7.6	11.39	0.86	7231.56
20	12.5	10.74	1.51	5052.42
21	10.8	10.97	1.28	5614.48
22	12.2	10.78	1.47	5144.92
23	10.1	11.07	1.18	5859.75
24	3.9	11.83	0.42	12150.60

Evaluation of Thevenin equivalent hydraulic network for extended period simulations

For the extended period simulations, three demand patterns are generated with the demand factors as shown in Table 3 for a time period of 24 hours. Here, the hydraulic time step is considered as 1 hour and the reporting time step is fixed at 15 minutes. To evaluate the equivalent hydraulic network obtained using the Thevenin theorem, the total demand in the sub-network is varied from 2×10^{-3} m^3/s to 6×10^{-3} m^3/s with a step size of 1×10^{-3} m^3/s (case 1 to case 6 in Table 2). In each case, the total sub-network demand is distributed equally among all the nodes of the sub-network. The demand Pattern 3 in Table 3 is assigned to all the main network nodes as the base demands. In the sub-network, the odd numbered nodes are assigned with the demand Pattern 1 and the even numbered nodes are assigned with the Pattern 2 for demonstration purpose.

The solutions of the full network (i.e. sub-network connected with the original main network) are considered as the reference solutions for evaluating the accuracy of the reduced network. The flow solutions of the reduced network are found to be exactly in agreement ($|\Delta Q| = 0$ m^3/s) with those of the reference solutions for all the cases. The sub-network node N40 is found to have the maximum variation in head values for the changes in the sub-network demands. Therefore, N40 is chosen for further comparison. Figure 13 compares the head values at N40 of the sub-network at different time periods for all the cases. From this figure, the reduced network is giving almost the same results to that of the reference solutions. The maximum percentage error in the head value in all the cases is less than 1%. The reduced network shown in Figure 11 simulates approximately ten times faster than the original network in Figure 10. Since the system curve approach is found to be less accurate than the Thevenin equivalent approach, the analysis by the system curve approach is avoided for this application to limit the length of the paper.

Table 6. Comaparision of heads at N34.

Time (Hrs)	Original (m)	Thevenin (m)	System curve (m)	% Error Thevenin	% Error System curve
1	8.40	8.37	8.35	0.36	0.60
2	9.15	9.09	9.21	0.66	0.66
3	9.03	8.96	9.10	0.78	0.78
4	9.70	9.59	9.85	1.13	1.55
5	6.05	6.06	5.73	0.17	5.29
6	5.13	5.14	4.70	0.19	8.38
7	5.13	5.14	4.70	0.19	8.38
8	5.62	5.62	5.33	0.00	5.16
9	7.17	7.15	7.07	0.28	1.39
10	7.17	7.15	7.07	0.28	1.39
11	7.97	7.93	7.95	0.50	0.25
12	8.72	8.65	8.80	0.80	0.92
13	7.17	7.15	7.07	0.28	1.39
14	7.54	7.52	7.42	0.27	1.59
15	10.56	10.36	10.76	1.89	1.89
16	5.98	5.98	5.58	0.00	6.69
17	9.03	8.96	9.10	0.78	0.78
18	8.17	8.12	8.19	0.61	0.24
19	8.90	8.81	9.01	1.01	1.24
20	5.38	5.39	5.02	0.19	6.69
21	6.96	6.95	6.80	0.14	2.30
22	6.05	6.06	5.73	0.17	5.29
23	7.92	7.89	7.84	0.38	1.01
24	11.24	10.97	11.44	2.40	1.78

From the two examples presented above, one can conclude that the proposed network-reduction method using the Thevenin theorem for electrical circuits can be efficiently used to analyse an extended network connected to a large existing network system or a relatively smaller part of a large network system within a range of realistic demand factors. It is also found that as the size of the main network increases, the accuracy of the proposed method appears to improve and corresponding computational efficiency becomes more evident. The proposed network reduction method is unique compared to the conventional network reduction methods and it is presented for the first time in this study. The significant advantage in computation time can be observed while analysing a small network connected with a large network unlike the existing network reduction methods. Table 7 summarizes the possible reduction in computation time compared to some of the existing similar network reduction methods (Tao et al. 2009; Perelman and Ostfeld 2011; Jiang et al. 2013; Di Nardo et al. 2018) by considering only the number of nodes in the reduced network. Unlike other network reduction methods, the proposed method has only two elements in the reduced network irrespective of the size of the existing network; hence, a significant reduction in computation time is obvious. The computation overheads of the proposed method can also be minimized if a hydraulic engine is developed instead of using multiple packages such as EPANET and LTspice. Nevertheless, the proposed network reduction

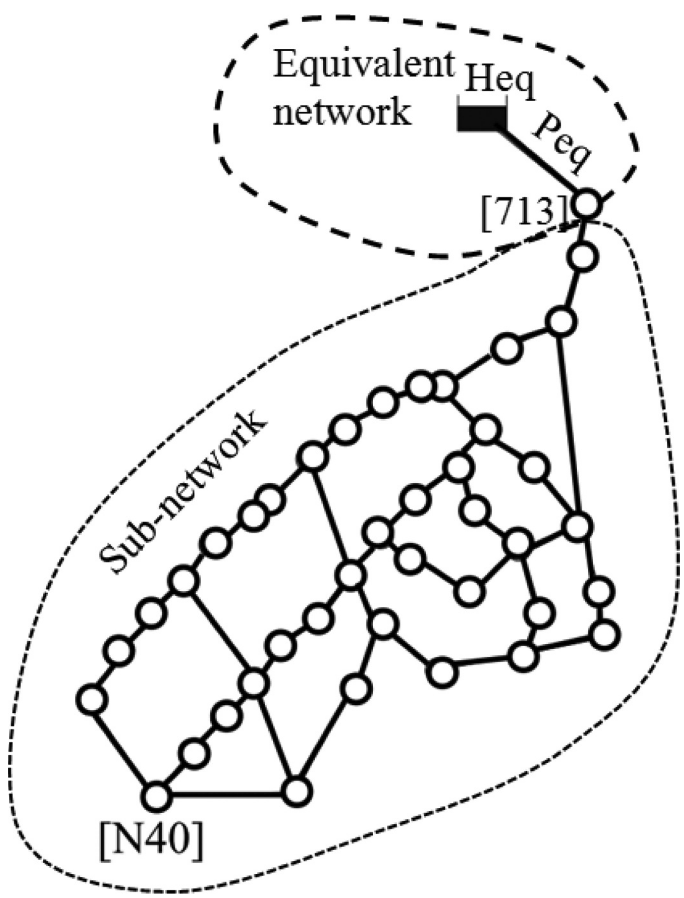


Figure 11. Equivalent pipe network for the original network shown in Figure 10.

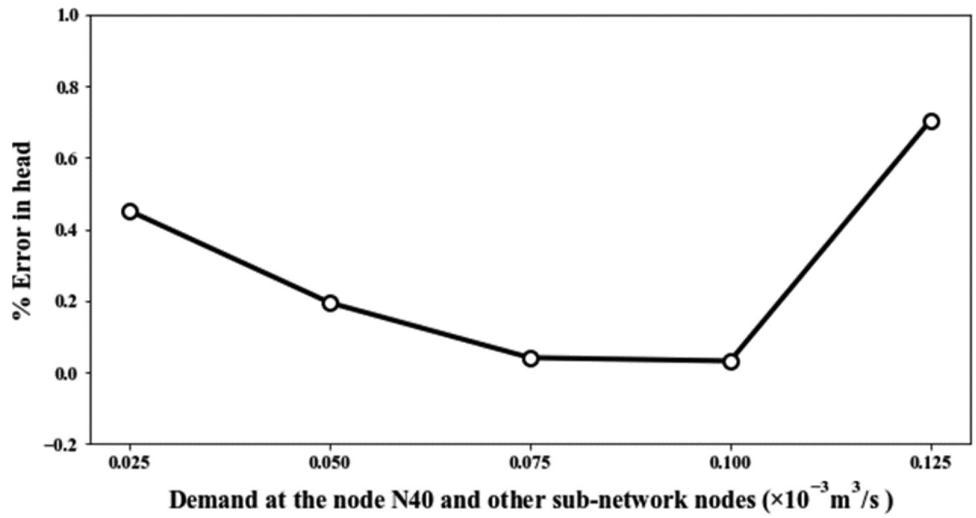


Figure 12. Analysis of sub-network with variable demands: percentage error in heads at the same node as obtained from the Thevenin network in comparison with the original nonlinear network.

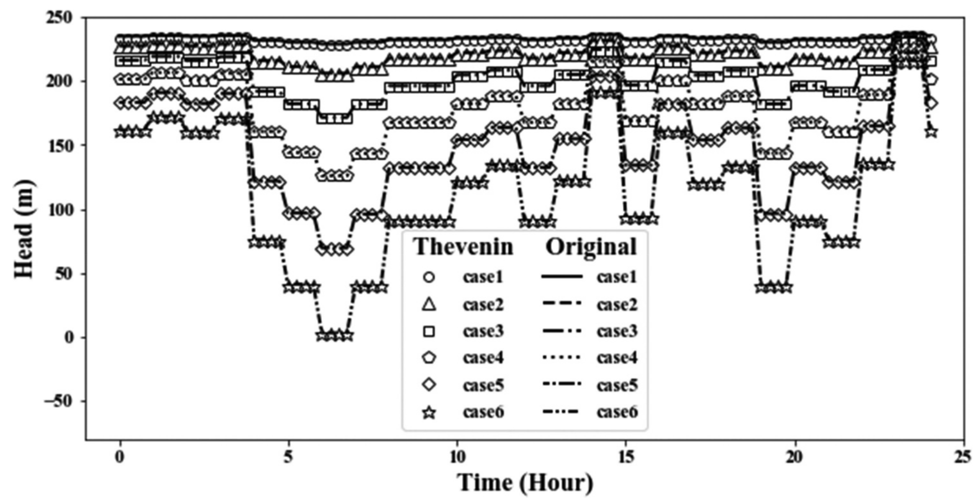


Figure 13. Extended period analysis: comparison of heads at N40 of additional network.

Table 7. Comparison of the proposed method with different WDN simplification methods.

		Case study: number of elements in		
Reference	Network reduction method	Main network	Reduced network	% Reduction in number of elements
Tao et al. (2009)	Skeletonization	7378	1247	83
Martínez-Alzamora, Ulanicki, and Salomons (2014)	Gaussian elimination	3733	1477	60
Jiang et al. (2013)	Branch collapsing, series and parallel pipe merging	48660	9901	79
Di Nardo et al. (2018)	Identification of a primary Network	284	195	31
Present work	Thevenin theorem	942 (variable)	2 (fixed)	99

method, therefore, opens up an excellent opportunity to efficiently analyze a WDN for stochastic demands under various hydraulic and operating constraints.

Scope and limitations

The proposed network reduction method can be used for analyzing a small portion of a large network or when a large network is extended by adding a sub-network. The proposed method requires only three simulations to derive the equivalent parameters irrespective of the size of the network. One EPANET simulation for finding the head loss and flow values in each pipe. Then the network is linearized, and the netlist is created for LTspice simulation. Two electrical simulations are required to find open circuit voltage and short circuit current using the netlist. These simulations require less computation time, and also, this is a one-time operation. The significant reduction in computation time can be exploited to efficiently study the reliability and optimization of a network for constraints such as stochastic demands, different layout configurations or for different properties of hydraulic elements. Though the accuracy and computational efficiency of the proposed network reduction method are demonstrated using the freely available pipe flow simulation software EPANET, the methodology can be easily implemented in any other pipe flow simulators. However, the proposed network reduction method suffers from limitations such as: (a) the analysis can only be focused on a particular portion of the network. If required, any changes in the remaining (upstream) part of the network cannot be directly addressed by the Thevenin equivalent network. In that case, a reorganization of that part of the network must be carried out separately before the application of the Thevenin theorem. Further research may solve this issue more efficiently; (b) the sub-network required to be connected to the main network through only one pipe for applying the single-port Thevenin theorem as presented in this study. However, the authors are working on the application of the multi-port Thevenin theorem (Corazza, Someda, and Longo 1969) to hydraulic networks and this can resolve such a limitation; (c) the linearization of the network is necessary for applying Thevenin theorem and (d) the circuit simulator LTspice is required to be used in the absence of a similar hydraulic simulator and scientific community can invest effort in this direction once the advantages of the Thevenin theorem for WDN analysis is realized. An independent computer code based on the proposed concept can make different operations systematic and computational overheads can be eliminated. However, a simplified system curve-based approach derived from the understanding of the Thevenin theorem can also be an alternative solution provided the accuracy is acceptable.

Conclusions

For the first time, the application of the Thevenin theorem for electrical circuits is demonstrated to significantly reduce a hydraulic pipe network for the purpose of analyzing the expansion of an existing network. Unlike many other network reduction methods, the proposed methodology can reduce a large pipe network into a simple network having a single source reservoir and an equivalent pipe. The reduced simple network can be directly connected to a sub-network and analysed for many possible sub-network demands. The simulation results from the reduced networks corresponding to two sizable hydraulic networks are highly accurate within a range of realistic demand variations in the connected sub-network. The applications are presented for both steady and extended period simulations. The proposed network reduction methodology is shown to consume significantly less CPU time as the size of the main network increases. The proposed method is qualitatively compared with other existing WDN reduction methods in terms of the number of nodes in the reduced network. A simplified system curve-based approach requiring no network linearization is also demonstrated for extended period simulations. However, such a simplified method is comparatively less accurate but may provide some quick insight. Hence, this work is presented as an efficient network reduction tool for replacing a large pipe network system with a simplified network without compromising the accuracy of the stated variables.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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