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Probabilistic capacity models for corroding strands in Post-Tensioned bridges with voided tendons

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ABSTRACT: The presence of unwanted air-voids in the grouted tendons and the resulting strand corrosion can be a serious threat to the long-term structural performance of post-tensioned (PT) bridge systems. Because of these potential corrosion issues, bridge management authorities are in dire need of probabilistic models to assess the time-variant structural reliability of PT systems. Because the strands play a major role in the load carrying capacity, probabilistic strand capacity models are necessary to assess the structural reliability of PT systems. Such models can help in making decisions on repair and maintenance programs and in assuring long-term structural safety. This paper presents the development of probabilistic strand capacity models and its assessment using the data on remaining tension capacity from a one-year strand corrosion test program (with a sample size of 209 test specimens). The experimental design for the test program includes independent variables such as grout types, void types, oxygen concentration, carbon dioxide concentration, chloride ion concentration, and relative humidity conditions that are intended to represent the possible range of field conditions. A diagnostic study is conducted to investigate the nature and degree of influence of each independent variable on the strand capacity. Then two probabilistic models that relate the strand capacity to the significant independent variables are constructed. The two probabilistic models are assessed using a Bayesian approach. The Models A and B have mean absolute percentage errors ($MAPE_C$) equal to 2.79 and 2.88%, respectively, indicating reasonably good predictions of strand capacities. The developed capacity models are unbiased and account for the prevailing uncertainties, including model errors, arising from an inaccurate model form or missing variables, measurement errors, and statistical uncertainty.

1 INRODUCTION

In the US, the construction of long span, post-tensioned (PT) segmental bridges began in the1960s. Until recently, these PT structures were thought to be highly durable and resistant to corrosion. However, recent inspections conducted by various federal and state transportation agencies reported the presence of air-voids in the grouted tendons and resulting strand corrosion (NCHRP 1998, FDOT 2001). According to Schupack (2004) and others, evaporation of bleed-water, poor grouting and/or poor construction practices are possible reasons for this unwanted void formation inside the tendons. Under various corrosive field conditions, such as seawater, salt-fog, and/or de-icing /anti-icing salts, the strands at these void locations can have a higher probability of corrosion, especially localized corrosion, resulting in a reduced tension capacity (capacity herein).

The unwanted air-voids inside the PT ducts (i.e., tendons) can create long-term performance issues and can adversely affect the structural capacity and reliability of these transportation structures. According to Poston et al. (2003), a 75% strand strength reduction can result in 50% or more reduction in live load carrying capacity of the bridge. Moreover, structural failures have been reported due to the increased level of strand corrosion in grouted tendons having air-voids that are exposed to corrosive environments (NCHRP 1998, FDOT 2001). These studies and field observations indicate that there is a need to develop a time-variant structural reliability model and evaluate the long-term performance characteristics of these bridge structures.

A first step towards developing such a structural reliability model is to develop probabilistic capacity models for corroding strands. A one-year strand corrosion test program is conducted to generate necessary data to formulate and assess the probabilistic capacity models. In this study, 0.6 inch diameter, low relaxation, 7-wire strands meeting the requirements of "*Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete*" (ASTM A416/A416M-06) are used.

Table 1. Number of test specimens for each combination of the *qualitative variables*.

Sample group	Grout class	Void type: Orthogonal	Parallel	Inclined	Bleed water
1	A	3	3	3	3
	C	3	3	3	3
2	A	3	3	3	3
	C	3	3	3	3
3	A	3	3	3	3
	C	3	3	3	3
4	A	5	5	5	5
	C	5	5	5	5
5	A	5	5	5	5
	C	5	5	5	5
6	A	5	5	5	–
	C	5	5	5	–
7	A	3	3	5	5
	C	5	5	5	5

Table 2. Number of test specimens and exposure levels in each combination of *quantitative variables*.

Sample group	Number of text specimens	Level of concurrent exposure conditions: % O_2	% CO_2	% Cl^-	% RH
1	24	15	0.03	0.0001	50
2	24	21	0.03	0.0001	50
3	24	21	3	0.0001	50
4	40	10	0.015	0.006	100
5	40	10	0.015	0.018	100
6	30	10	0.015	0.18	100
7	40	10	0.015	1.8	100

2 EXPERIMENTAL DESIGN

An experimental program with strand corrosion tests is designed to obtain data for developing the probabilistic strand capacity model. The experimental design includes grout type and void type as *qualitative variables* and various environmental exposure conditions as *quantitative variables*. The *quantitative variables* were oxygen concentration (O_2), carbon dioxide concentration (CO_2), and chloride ion concentration (Cl^-), and relative humidity (RH) conditions. It should be noted that every test specimen is concurrently exposed to a particular level of each of these exposure conditions. The experimental design information on the *qualitative* and *quantitative variables* is provided in Tables 1 and 2, respectively. The total sample size is 209 test specimens.

2.1 *Grout types*

The Post-Tensioning Institute (PTI) classifies grout materials into four different classes based on material specifications and field requirements (PTI 2003). Most of the existing bridge structures have been constructed using Class A grout. Class A grout consists of Portland cement and water only. Under similar conditions, the Class A grout can have higher probability of void formation when compared to the pre-packaged Class C grout with good fresh characteristics. Hence, Class C grout is being used in newly constructed PT bridge structures. In general, Class A and Class C grouts have been used in most PT bridges in the USA. In this test program, Class A grout with 0.44 w/c and Class C grout with 0.27 w/c have been used as two factors for the *qualitative variable*, grout type.

2.2 *Void types*

The characteristics of various types of voids obs5erved in PT systems can depend on fresh characteristics of grout material, grouting techniques, tendon profiles, design considerations, and other factors. This test program considers four types of voids for the *qualitative variable*, void type. A void can be of orthogonal, parallel, inclined, and bleedwater type. These void types are intended to mimic the possible field void conditions, which include the local angle between the grout surface and longitudinal axis of the partially embedded strand specimen and typical bleedwater conditions. These void types can have a varying impact on localized strand corrosion. Environmental exposure conditions can also influence the strand corrosion.

2.3 *Environmental exposure conditions*

2.3.1 *Control conditions*

To facilitate a comparative study, a group of samples with control exposure conditions were tested. In Tables 1 and 2, these samples are shown as Group 1 samples with concurrent exposure conditions such as 15% O_2, 0.03% CO_2, 0.0001% Cl^- and 50% RH.

2.3.2 *Oxygen concentrations (O_2)*

The presence of O_2 is necessary for the occurrence of electrochemical corrosion of steel embedded in cementitious materials (ACI 2001). Hence, O_2 was included in the experimental design as a quantitative variable to investigate the strand corrosion rate as a function of O_2 level. In general, the normal atmospheric oxygen level is 21%. However, because box girders can be considered as confined spaces and the steel tendons are encased by high density polyethylene (HDPE) ducts with low air permeability, the level of oxygen to which the strands are exposed to can be lower than 21%. As shown in Table 2, the three O_2 levels considered in this test program are 10%, 15%, and 21%.

2.3.3 *Carbon dioxide concentrations (CO_2)*

When carbonation occurs, the pH of the cementitious material can drop from between 12.6 to 13.5 to as low as 9 (Neville 1998). Because of this drop in pH of the cementitious material, the embedded steel reinforcement actively corrodes. Hence, the corrosion rate of strands, when exposed to carbonated grout, is important to predict the time-variant structural reliability. According to ACI (2001), the CO_2 level under normal atmospheric conditions is 0.03%. However, the CO_2 level inside the PT box girders may vary due to the exhaust gases from vehicles and/or industrial chimneys. This indicates that a PT bridge located in an urban and/or industrial area can have higher CO_2 exposure than one located in a rural area. Another case to be considered is the situation when the high relative humidity or moisture exists inside the tendons. In this case, the concentration of CO_2 is expected to be lower than normal atmospheric conditions. As shown in Table 2, the three levels of CO_2 considered in this experimental program are 0.015%, 0.03%, and 3%.

2.3.4 *Chloride ion concentrations (Cl^-)*

Most of the corrosion issues in transportation structures are due to the presence of Cl^- environment. The Cl^- acts as a catalyst for the electrochemical corrosion of steel embedded in cementitious material. The five different Cl^- concentrations to which the strand corrosion samples are exposed to are 0.0001%, 0.006%, 0.018%, 0.18%, and 1.8%. A concentration of 0.0001% Cl^- is obtained by exposing the samples to normal room conditions with 50% *RH*. Standard water from laboratory faucets or taps is used as 0.006% Cl^- solution. The 0.018%, 0.18%, and 1.8% Cl^- solutions are prepared by mixing the appropriate amount of crystalline sodium chloride with the water.

2.3.5 *Relative humidity conditions (RH)*

Like O_2, *RH* (or moisture) is an essential component for electrochemical corrosion of steel embedded in cementitious materials (ACI 2001). In external PT systems, the strands are embedded in grout materials encased by HDPE ducts. Water or moisture can penetrate into the duct through broken locations and other access points on the HDPE duct wall or anchorage zones and then reach strands. Once collected inside these tendons, it is very difficult for the moisture to evaporate or dry, resulting in high *RH*. High *RH* can propagate the strand corrosion process. Hence *RH* is another critical parameter affecting the strand corrosion in PT systems. In this test program, the two levels of *RH* considered are 50% and 100%.

3 STATISTICAL MODELING

As a first step in modeling the tension capacity of strands, diagnostic plots are developed to explore the effect of each *qualitative* and *quantitative variable*. Then using an engineering understanding of the corrosion process and mathematical principles, the variables are classified as "critical" and "non-critical" variables. Then two probabilistic models (Model A and Model B) are formulated using the critical variables.

3.1 *Critical variables*

The diagnostic plots indicate that the void type, chloride level, and relative humidity are important variables affecting the tension capacity. The possible relationships between these variables and the tension capacity are as follows.

- Void type: The strand specimens with horizontal void type exhibit less reduction in capacity than those with orthogonal, inclined, or bleedwater void types. However, because of the actual length of the tendons in PT bridges (say, several miles) and other technical difficulties associated with the bridge inspection procedures, it might be difficult to identify the type of existing voids in PT systems. Hence, though the void type is a critical variable, the research team decided to develop models that do not distinguish between void types and can be applied without knowing the void type. As such, this variable will not be explicitly shown in the probabilistic model forms that are provided later in this paper.
- Chloride ion concentration: The diagnostic plot, as shown in Figure 1a, indicates a possible power or reciprocal relationship between capacity and chloride ion concentration. Some data points in the left region of Figure 1a are clustered and it is difficult to distinguish between their Cl^- levels. Hence, an additional plot with a different abscissa scale is provided in Figure 1b for clarity. The data points in Figure 1b correspond to 0.0001%, 0.006%, 0.018%, and 0.18% Cl^- concentrations. In this paper, two independent models are formulated to explore the appropriateness of power and/or reciprocal relationships.
- Relative humidity: Consider the diagnostic plot shown in Figure 2. Because data corresponding to only two levels of *RH* is available, any functional relationship can be assumed between capacity and *RH* level. However, in this paper, a linear relationship is chosen to develop a simple model. This simple linear relation is shown using the inclined-dotted line with a negative slope connecting the means of observed capacities. The hatched region in Figure 2 indicates the region below the nominal capacity (i.e., 260 kN (58.6 kips)), as provided by the strand manufacturer. This value corresponds

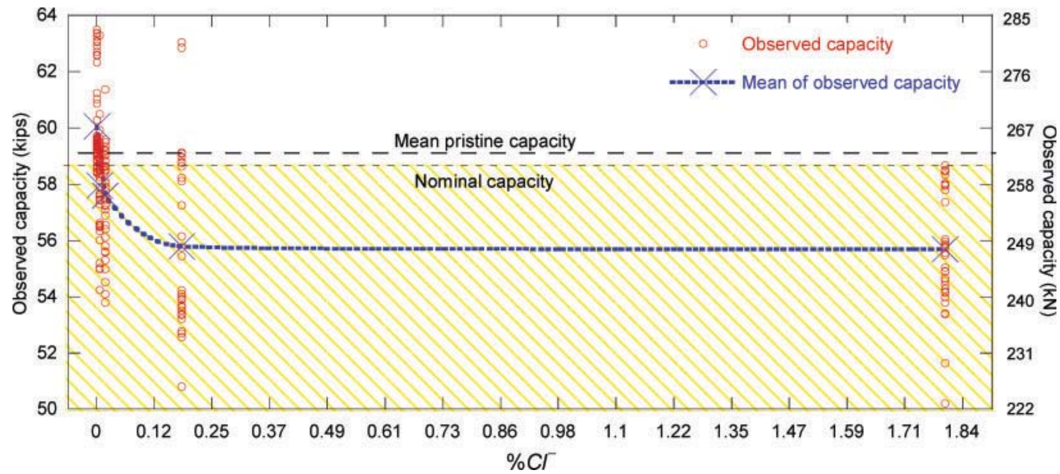

Figure 1a. Observed capacity as a function of chloride level.

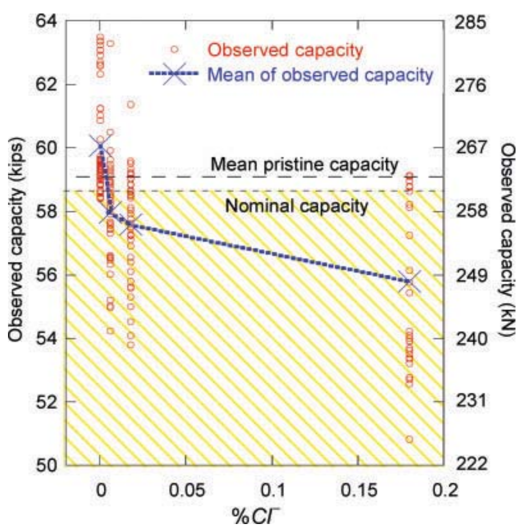

Figure 1b. Observed capacity as a function of chloride level (Zoomed in area from the left region of Figure 1a).

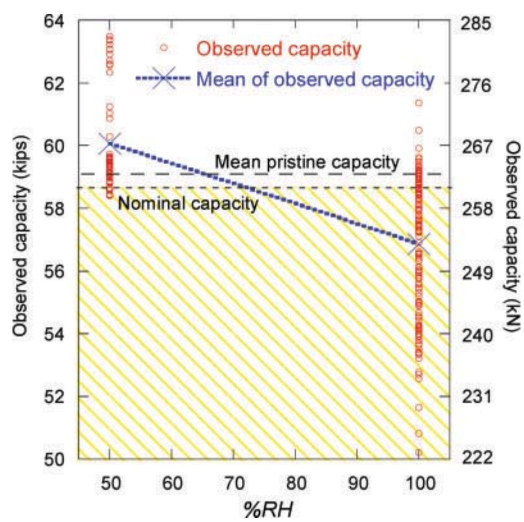

Figure 2. Observed capacity as a function of relative humidity.

to an axial stress of 270 ksi. A horizontal-dashed line indicating the mean capacity of pristine strands tested in this study (i.e., 262 kN (59 kips)) is also shown.

It should be noted that the mean of the observed or measured capacities of pristine strands (i.e., mean pristine capacity) is slightly higher than the nominal capacity. However, after one year of corrosive exposure in the laboratory, the true capacity of many strand samples dropped to below nominal capacity. Hence, after a field exposure period equivalent to one year of laboratory exposure, one cannot conservatively use the nominal capacity values while performing condition assessment of bridges. This shows the need for a probabilistic model to estimate the actual capacity of strands exposed to corrosive conditions.

Prior to the discussion on the probabilistic model formulations using the critical variables, a brief discussion of the non-critical variables is provided.

3.1.1 *Non-critical variables*

The diagnostic study indicates that after one year of laboratory exposure the grout type, O_2 concentration, and CO_2 concentration are not critical variables influencing the strand capacity.

- Grout type: Both Class A and Class C grouts exhibited similar corrosion effects on tension capacity. Hence, this variable is not considered for probabilistic model formulation. However, it should be noted that the grout type can have a significant influence on the number of voids formed in a PT system. This probability of void formation should also be considered when assessing the reliability of PT bridges.
- Carbon dioxide: Though the level of CO_2 is increased by 100 times, no significant reduction in capacity is observed. Therefore, this quantitative variable is not considered herein.
- Oxygen: No significant reduction in capacity is observed for different O_2 levels. Hence, this quantitative variable is not considered while formulating the probabilistic capacity model.

3.2 *Probabilistic capacity models*

Following Gardoni et al. (2002), a general probabilistic capacity model is formulated as follows:

$$R_C(\mathbf{x}, t, \Theta) = \gamma(\mathbf{x}, t, \boldsymbol{\theta}) + \sigma\varepsilon \tag{1}$$

where, $R_C(\mathbf{x}, t, \Theta)$ is the ratio between the actual strand capacity, $C(\mathbf{x}, t, \Theta)$, and the nominal strand capacity, $\hat{c}$; $\mathbf{x}$ is the vector of independent variables (i.e., Cl^- and RH); t is the time function corresponding to the observed strand capacity; and the vector $\Theta = (\theta, \sigma)$ represents the set of unknown parameters, where $\theta = (\theta_1, \ldots, \theta_k)$ is a $k \times 1$ vector; k is the number of regression parameters̃. The term $\gamma(\mathbf{x}, t, \theta)$ is a capacity correction function; and $\sigma\varepsilon$ is the model error term, where σ represents the standard deviation of the model error and ε is a random variable with zero mean and unit standard deviation. Note that for given $\mathbf{x}$, t, θ and σ, the variance of the model, Var[$R_C(\mathbf{x}, t, \Theta)$], is equal to σ^2.

The correction function, $\gamma(\mathbf{x}, t, \theta)$, captures two reasons why the actual capacity might be different from the nominal one: (1) the actual capacity of a pristine strand is typically greater than the nominal capacity, and (2) corrosion might lead to a reduction in capacity.

The proposed model is dimensionless. As a result, the parameters θ_j are also dimensionless. Diagnostic plots of the residuals and the observed capacities against predicted capacities or individual regressors verified the correctness of the following two assumptions: (1) the model variances σ^2 is independent of $\mathbf{x}$ (homoskedasticity assumption), and (2) ε has the normal distribution (normality assumption).

The next section develops two capacity models (Model A and Model B) based on the observations made on the diagnostic plots and the model formulation provided in Eq. (1).

3.2.1 *Model A*

As mentioned earlier, Figure 2 indicates that a linear relationship could be used to express the dependency of the strand capacity on RH. Figures 1a and 1b shows evidence of either a power or a reciprocal relationship between the capacity and the Cl^- level.

Model A is constructed using a power relationship. To produce a linear regression model, the Cl^- level is transformed by taking $\log_{10}(Cl^-)$. The resulting probabilistic model can be expressed as follows:

$$R_C(\mathbf{x}, t, \Theta) = \theta_1 + \theta_2 \log_{10}(Cl^-) + \theta_3 RH + \sigma\varepsilon \tag{2}$$

It should be noted that the term t is not explicitly shown in the right hand side of the Eq. (2). This is because $t = 1$ year for the data set studied in this paper.

3.2.2 *Model B*

Model B still uses the linear dependency between the strand capacity and RH, but it explores the reciprocal relationship between the capacity and the Cl^- level. In this case to produce a linear regression model, the Cl^- level is transformed by taking its reciprocal, $1/Cl^-$. The resulting probabilistic model can be expressed as follows:

$$R_C(\mathbf{x}, t, \Theta) = \theta_1 + \theta_2 \left(\frac{1}{Cl^-}\right) + \theta_3 RH + \sigma\varepsilon \tag{3}$$

Here also, because the specimens were exposed to corrosive environments only for one year (i.e., $t = 1$), the term t is not shown in the right hand side of the Eq. (3).

3.3 *Assessment of probabilistic capacity models*

Given that

- The proposed probabilistic models (Eqs. (2) and (3)) are linear in parameters,
- No prior information is available about the distribution of parameters, and
- No upper or lower bound data are used,

a closed-form solution can be used to assess the models. The various steps in this closed-form solution procedure are summarized next.

The linear probabilistic models, Model A and B can be re-written in the following form:

$$\mathbf{R}_C = \mathbf{G}\boldsymbol{\theta} + \sigma\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{R}_C$ is an $n \times 1$ vector of capacity observations, $(R_{C1}, \ldots, R_{Cn})$; n is the number of observations or sample size (in this paper $n = 209$); $\mathbf{G}$ is an $n \times k$ matrix of known regressors; k is the number of regression parameters (in this study, $k = 3$); ε is an $n \times 1$ vector of random variables having zero mean and unit variance. The remaining quantities are as defined in Eq. (1). As provided in Box and Tiao (1992), the posterior distribution of the unknown model parameters Θ can then be written as follows:

$$p(\Theta|\mathbf{R}_C) \propto p(\Theta)\, p(s^2|\sigma^2)\, p(\hat{\theta}|\theta,\sigma^2)$$

where,

$$\begin{aligned}
\hat{\boldsymbol{\theta}} &= (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{R}_C \\
s^2 &= \left(\frac{1}{v}\right)(\mathbf{R}_C - \hat{\mathbf{R}}_C)'(\mathbf{R}_C - \hat{\mathbf{R}}_C) \\
v &= n - k \\
\hat{\mathbf{R}}_C &= \mathbf{G}\hat{\boldsymbol{\theta}}
\end{aligned} \tag{5}$$

Using a non-informative prior distribution for θ and by assuming θ and $\log(\sigma)$ to be locally uniform and approximately independent, $p(\Theta)$can be expressed as follows:

$$p(\Theta) = p(\boldsymbol{\theta})\, p(\sigma^2) \propto \sigma^{-2} \tag{6}$$

The joint posterior distribution, $p(\Theta|\mathbf{R}_C)$, in Eq. (5), can be rewritten as follows:

$$p(\boldsymbol{\theta},\sigma^2|\mathbf{R}_C) \propto p(s^2|\sigma^2)\, p(\boldsymbol{\theta}|\hat{\boldsymbol{\theta}},\sigma^2) \tag{7}$$

Furthermore, the marginal posterior distribution of σ^2 is $vs^2\chi_v^{-2}$, where χ_v^{-2} is the inverted chi-square distribution with v degrees of freedom, with mean $vs^2/(v-2)$ and variance $2v^2s^4/[(v-2)^2(v-4)]$.

The marginal posterior distribution of θ is:

$$p(\boldsymbol{\theta}|\mathbf{R}_C) = \left[\frac{\Gamma\left(\frac{v+k}{2}\right)\cdot|\mathbf{G}'\mathbf{G}|^{1/2} s^{-k}}{\left[\Gamma\left(\frac{1}{2}\right)\right]^k \Gamma\left(\frac{v}{2}\right)\left(\sqrt{v}\right)^k}\right]\left[1 + \frac{(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})'\mathbf{G}'\mathbf{G}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})}{vs^2}\right]^{-(v+k)/2} \tag{8}$$

where, $-\infty < \theta < \infty$, and $i = 1, \ldots, 3$

This follows a multivariate t distribution, $t_k[\hat{\theta}, s^2(\mathbf{G}'\mathbf{G})^{-1}, v]$, where $\hat{\boldsymbol{\theta}}$ is both the mode and the mean of θ and $vs^2(\mathbf{G}'\mathbf{G})^{-1}/(v-2)$ is the covariance matrix of θ.

4 RESULTS AND DISCUSSIONS

The posterior statistics for the unknown model parameters θ are assessed using the closed-form solution provided in Section 3.3 and are summarized in Tables 3 and 4. In addition, for Model A, the posterior mean and standard deviation of σ are 0.0351 and 0.0111, respectively. For Model B, the posterior mean and standard deviation of σ are determined to be 0.0355 and 0.0112, respectively.

Table 3. Posterior statistics of unknown parameters (Model A).

Parameters	Mean	Standard deviation	Correlation coefficients θ_1	θ_2	θ_3
θ_1	0.9625	0.0226	1		
θ_2	−0.0172	0.0030	0.91	1	
θ_3	−0.0001	0.0002	−0.98	−0.85	1

Table 4. Posterior statistics of unknown parameters (Model B).

Parameters	Mean	Standard deviation	Correlation coefficients θ_1	θ_2	θ_3
θ_1	−3.4042	0.8714	1		
θ_2	0.0002	0.000044	−0.99	1	
θ_3	0.0436	0.0087	−1	0.99	1

The high values of the correlation coefficients are induced by the particular combinations of Cl^- and RH levels used in the experimental program. The experimental design is such that only low levels of Cl^- are combined with low levels of RH and only high levels of Cl^- are combined with high levels of RH. These combinations induced spurious high correlations. In addition, there are difficulties in interpreting the numerical values of empirical parameters, θ_i, especially in the case of high positive or negative correlations. To obtain more accurate estimates of the correlation coefficients, additional experiments should be conducted with [low Cl^- $\leftrightarrow$ high RH] and [high Cl^- $\leftrightarrow$ low RH] combinations. The exhibited multicollinearity problem can possibly be resolved by performing ridge regression or principal component regression analysis. However, this analysis is not provided in this paper.

The posterior mean of the standard deviation of the model error, σ, can be used to evaluate the accuracy of the two models. To provide a more intuitive measure of the model accuracy, the mean absolute percentage error (MAPE_C) is also calculated for the two probabilistic models presented. The MAPE_C can be seen as the average error in the model expressed as a percent of the measured value. $MAPE_C$ can be mathematically expressed as follows:

$$MAPE_C = \frac{1}{n}\left[\sum_{i=1}^{n}\left(\frac{\left|E[C(\mathbf{x}_i,\boldsymbol{\theta})]-C_i\right|}{C_i}\right)\right]\times 100 \tag{9}$$

where C_i is the observed strand capacity and $E[C(\mathbf{x}_i,\theta)] = \hat{c}\gamma\,(\mathbf{x}_i, t, \hat{\theta})$ is the mean predicted capacity. The $MAPE_C$ values for Models A and B are 2.79% and 2.88%, respectively, indicating that both models can generate reasonably good and comparable results. Model A, is marginally more accurate as can be noted also by comparing the posterior means of σ. Additionally, $MAPE_{\hat{c}}$ is defined as follows:

$$MAPE_{\hat{c}} = \frac{1}{n}\left[\sum_{i=1}^{n}\left(\frac{\left|\hat{c}-C_i\right|}{C_i}\right)\right]\times 100 \tag{10}$$

This is obtained by replacing the mean predicted capacities in Eq. (9) with the nominal capacity, $\hat{c}$. It is found that the $MAPE_{\hat{c}}$ is 3.58%. Note that $MAPE_C < MAPE_{\hat{c}}$, indicating a larger scatter around the true capacity. The developed probabilistic models are unbiased (the term $\gamma\,(\mathbf{x}, t, \theta)$ corrects for the systematic error in estimating the actual capacity based only on $\hat{c}$) and are more accurate than $\hat{c}$, especially when severe corrosive conditions exist.

Figures 3 and 4 show the comparisons between the observed and predicted capacities for Models A and B, respectively. For a perfect model and a perfect experimental data set, the predicted and observed capacity should line up along the 1:1 solid line in Figures 3 and 4. However, in the present study, variability exists in the data obtained from the strand corrosion samples with identical exposure conditions. Such variability is both *epistemic* and *aleatory* in nature. The epistemic component can be reduced by improving the experimental procedures. The *aleatory* component could be associated with the material non-homogeneity and other unknown factors. Hence, the *aleatory* component cannot be reduced using *practical* engineering methods. The dashed-parallel-inclined lines indicate the region within one standard deviation from the 1:1 solid line. The hatched region indicates the region with capacities less than the nominal strand capacity supplied by the manufacturer, ($\hat{c} = 260$ kN (58.6 kips)).

A detailed analysis of the data in Figures 3 and 4 shows that with one year of laboratory exposure, the percentage of strands having capacities lower than nominal capacity increases with an increase in RH and Cl^- levels. In particular, the capacity of many strands exposed to 100% RH and Cl^- conditions have dropped to values below the nominal strand capacity. Moreover, for all void types, the capacity of strands exposed to 50% RH with 0.0001% Cl^- conditions does not drop below the nominal capacity indicating that high RH and the presence of Cl^- at a level $< 0.006\%$ can cause active corrosion of strands.

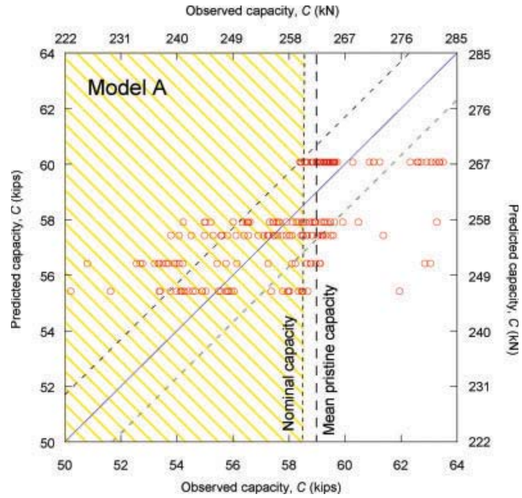

Figure 3. Comparison between the observed capacity and the capacity predicted using Model A.

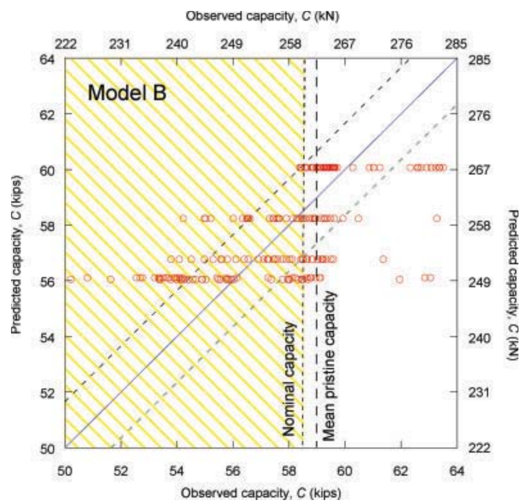

Figure 4. Comparison between the observed capacity and the capacity predicted using Model B.

5 LIMITATIONS OF THE PROBABILISTIC CAPACITY MODELS

It should be noted that some of the laboratory conditions are more severe than most field conditions. For this model to be used in the field, the variations in the field environmental conditions need to be considered and an appropriate time function needs to be developed. This is not included in the scope of this article. The currently developed probabilistic models use a unit value for the time variable (i.e., $t = 1$ year). However, time function is a crucial factor in determining the long-term structural performance of PT systems. Experiments with longer test durations are currently being conducted at Texas A&M University. They will provide the necessary data to model a time function.

6 CONCLUSIONS

Two probabilistic models are developed to estimate the actual capacity of 0.6 inch diameter, low relaxation, 7-wire strands that are used in PT bridges that may have voided tendons and corrosive environmental exposure conditions. It is concluded that after one year of laboratory exposure, oxygen (O_2) and carbon dioxide (CO_2) levels are not critical factors in affecting the actual capacity of the strands. On the other hand, the relative humidity (RH) and the chloride (Cl^-) exposure conditions are found to be critical in influencing the strand capacity. The developed probabilistic models are unbiased and provide a more accurate prediction of strand capacity than the nominal capacity, especially when severe corrosive conditions exist.

7 FUTURE WORK

An experimental program with longer test durations is currently underway at Texas A&M University to provide the data required for modeling a time function for laboratory exposure conditions. Also, methods are being explored to estimate the relationship between the laboratory exposure conditions and actual field conditions.

In most cases the identification of void types present in PT systems is a cumbersome and difficult task. Hence, the models *presented* in this paper are developed without distinguishing between the void types. However, a detailed bridge inspection may generate information on void types present. To facilitate better capacity prediction using the additional information on void type, independent probabilistic models for different void types are also being developed.

Ridge regression and/or principal component regression analysis will be performed to eliminate the multicollinearity problem exhibited by the parameters in the developed models.

The developed capacity models can be combined with probabilistic demand models to assess the vulnerability of existing PT bridges and to perform reliability based design of future PT bridges.

ACKNOWLEDGMENTS

This research was performed at Texas Transportation Institute, Texas A&M University, College Station, Texas through a sponsored project from the Texas Department of Transportation, Austin, Texas, whose support is highly appreciated.

REFERENCES

ASTM A416/A416M-06. 2006. Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete. *American Society of Testing and Materials, ASTM International*: West Conshohocken: PA.

ACI. 2001. ACI 222R-01: Protection of Metals in Concrete Against Corrosion. ACI Committee 222, Document No. 22201: *American Concrete Institute*: Farmington Hills: MI.

Box, G.E.P. and Tiao, G.C. 1992. Bayesian Inference in Statistical Analysis. 113–122. Addison-Wesley: Reading: MA.

FDOT. 2001. Mid-Bay Bridge Post-Tensioning Evaluation – Final Report. Florida Department of Transportation (FDOT). Corven Engineering, Inc. Tallahassee. FL.

Gardoni, P.G., Kiureghian, A.D. & Mosalam, K.M. 2002. Probabilistic Capacity Models and Fragility Estimates for Reinforced Concrete Columns based on Experimental Observations. *ASCE Journal of Engineering Mechanics* 128(10): 1024–1038.

NCHRP. 1998. Durability of Precast Segmental Bridges. (Editors: Poston, R.W. & Wouters, J.P.) NCHRP Web Document No. 15: Project 20-7/Task 92: National Cooperative Highway Research Program (NCHRP): Transportation Research Board: National Research Council: Washington: D.C.

Neville, A.M. 1998. Properties of Concrete. Fourth Edition, John Wiley & Sons: New York: NY: 498–499.

Poston, W.R; Frank, K.H.; & West, J.S. 2003. Enduring Strength. *Civil Engineering*: American Society of Civil Engineering: 73(9): 58–63.

Schupack, M. 2004. PT Grout: Bleedwater voids. *Concrete International*: 26(8): 69–77.

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